Sensitivity analysis of effective thermal conductivity of open-cell ceramic foams using a simplified model based on detailed structure

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ABSTRACT

The Effective Thermal Conductivity (ETC) of open-cell porous foams can be predicted from the detailed numerical simulation, considering the complex foam structure obtained from three-dimensional (3D) Computed Tomography (CT)-scan images. An alternative approach could be to consider simplified models for a quick and accurate estimation of the ETC. A model for ETC of open-cell porous foams, using such a simplified approach, has been proposed recently which relies upon a single numerical prediction of the dimensionless ETC under vacuum condition, evaluated using the detailed foam structure obtained from 3D CT-scan information. This model is applied in the present study in order to analyze the influence of different parameters, namely the microscopic porosity within the bulk solid material and the direction of heat transfer, on the ETC of open-cell ceramic foams. The present investigation demonstrates that the considered simplified modeling approach offers reasonable accuracy with reduced computational effort for the sensitivity analysis of ETC to different parameters.

INTRODUCTION

Porous foams are commonly used for a wide range of applications in numerous technological fields due to their special structural and thermo-physical properties. In this respect, the evaluation of their Effective Thermal Conductivity (ETC) has become a major requirement for the accurate prediction of heat transfer behavior of many systems, such as heat exchangers, thermal insulators, combustion systems, evaporators, etc., using homogenization approach [1-3]. The complex morphology of foam structures plays an important role in the overall heat transfer characteristics and hence the ETC can vary substantially among different foams of same material and porosity. Moreover, owing to the very high porosity of foams, total heat transfer can take place by both conduction as well as thermal radiation, particularly when the operating temperature as well as the applied temperature difference are relatively high [4]. The present investigation, however, is restricted only to the conduction heat transfer at room temperature, where the contribution of thermal radiation is seemingly small.

The ETC of porous foams can be numerically predicted from simulations of conduction heat transfer through them due to imposed temperature gradient by considering their detailed morphology. The structural information of foams, required by these simulations, nowadays can be obtained from the high resolution 3D CT-scan images. Such detailed approach is generally quite accurate, as well as time consuming. Therefore, in order to achieve a reasonable compromise between the accuracy and the computational effort, an alternative approach is to use simplified models for quick and accurate evaluation of the ETC, ideally presenting a wide range of applicability. A review on models for the ETC, presenting different levels of complexity, has been presented by Coquad et al. [5].

Recently, a model for evaluating the ETC of open-cell porous foams, using a simplified approach with one adjustable parameter, has been proposed by Mendes et al. [6]. This, parameter appearing in an explicit expression for the ETC, can be uniquely determined from a single numerical prediction of the dimensionless ETC under vacuum condition (made dimensionless with respect to the thermal conductivity of solid phase), based on the detailed geometry of foam. Quite obviously, evaluation of the dimensionless ETC under vacuum condition requires significantly reduced computational time, which scales at least to the order of porosity of foams and owing to the definition, the result is independent of the thermal conductivity of solid phase. Most importantly, as has been demonstrated by Mendes et al. [6], the dimensionless ETC under vacuum condition implicitly contains the relevant quantitative morphological
information of porous foams that is required for solving any further heat conduction problem.

During experiments, however, vacuum condition is extremely difficult to achieve, whereas conducting experiments with commonly available working fluids, like air or water, is rather straightforward. Hence the theory, proposed by Mendes et al. [6], required appropriate extension, which was successfully implemented by Mendes et al. [7, 8]. It may be noted here that although the model used by Mendes et al. [6-8] obeys limiting conditions for $k_f \to 0 \text{ and } k_f \to k_s$, certain amount of error of the order of 10% could be encountered for intermediate ranges of $k_f/k_s$, depending upon the morphology of porous foams. In order to eliminate this problem, very recently Mendes et al. [9] considered another simplified model for the ETC with two adjustable parameters, where the newly introduced parameter could be evaluated from the ETC of porous foams in presence of a low thermal conductivity working fluid, like air. With this modification, the maximum error was reduced to 2% for the entire range of $k_f/k_s$ for all investigated foam structures.

The present investigation deals with the prediction of ETC of open-cell ceramic foams using both detailed heat conduction model and simplified approach of Mendes et al. [6]. For both models, the detailed geometry of foams is obtained from 3D CT-scan images. In order to fix the threshold gray color for binarization of 3D CT-scan images into fluid and solid phases (voxels), the macroscopic porosity of foams, i.e. the porosity that can be captured by the CT voxel resolution, is required. This macroscopic porosity can be calculated from the total porosity of foams and the microscopic porosity within the bulk solid material. The total porosity, on the other hand, can be easily and accurately determined experimentally. However, accurate evaluation of the microscopic porosity is quite complex as well as time consuming and hence only a rough estimation can generally be obtained.

The major objective of the present study is to analyze the influence of microscopic porosity on the ETC of ceramic foams. Additionally, the degree of anisotropy of foams and its influence on the directional ETC is also investigated. Based on the comparison of results using detailed and simplified models, the validity and benefits of using the simplified modeling approach of Mendes et al. [6] for parametric studies on the ETC of open-cell porous foams are discussed. Therefore, the outcome of this study is expected to help researchers in analyzing the sensitivity of ETC to different parameters with reasonable accuracy and reduced computational effort, while relying upon the real morphology of open-cell porous foams.

**NOMENCLATURE**

- $q_{av}$: Average heat flux (W/m$^2$)
- $b$: model parameter
- $k$: Thermal conductivity (W/mK)
- $L_r$: Representative length of sample (m)
- $T$: Temperature (K)

**Greek Symbols**

- $\phi_m$: Microscopic porosity
- $\phi_M$: Macroscopic porosity
- $\phi_T$: Total porosity

**Subscripts**

- $A$: pure alumina
- $c$: cold
- $eff$: effective
- $f$: fluid phase
- $h$: hot
- $max$: maximum
- $min$: minimum
- $s$: solid phase

**Superscripts**

- $\sim$: dimensionless variable

**1 Ceramic foam samples**

Figure 1 shows an enlarged view of two different pure alumina foam samples, with 10 and 45 pores-per-inch (ppi), considered for the present study.

![Open-cell alumina foams](image)

Geometries of these foams are obtained from 3D CT-scan images with a resolution of 70 μm. For both foams, a similar total porosity $\phi_T = 0.89$ is determined gravimetrically by weighing and measuring dimensions (52.5 mm×52.5 mm×52.5 mm) of samples, considering the density of pure alumina to be 3984 kg/m$^3$ [10]. The representative size of these samples is chosen in such a manner that it is several times larger than their characteristic pore dimensions in order to allow accurate prediction of ETC. For present foams, the microscopic porosity $\phi_m$ of bulk alumina material, which could not
be captured by the CT voxel resolution, is estimated to be in the range of 5 to 20%, based on the information acquired from the foam producer. From total and microscopic porosities, the macroscopic porosity $\phi_M$ of foams can be calculated as:

$$\phi_M = \frac{\phi_T - \phi_m}{1 - \phi_m} \quad (1)$$

where $\phi_M$ is further used in order to fix the threshold gray color for binarization of 3D CT-scan images into fluid and solid voxels.

### 2 Prediction of ETC

In order to numerically determine the ETC of porous foams, they are considered to be formed by two distinct phases: a solid and a fluid phase, with thermal conductivities $k_s$ and $k_f$, respectively.

The ETC of a representative cubic sample of the open-cell ceramic foam in a particular direction (see Figure 2) is numerically predicted from the converged temperature field by applying the averaged Fourier's law of heat conduction as follows:

$$k_{\text{eff}} = \frac{q_{av} L_r}{T_h - T_c} \quad (2)$$

where $L_r$ is the distance between hot and cold boundaries those are maintained at temperatures $T_h$ and $T_c$, respectively. The remaining four boundaries of the computational domain are assumed to be adiabatic in order to ensure uni-directional heat transfer in an averaged sense. The average steady-state heat flux $q_{av}$, in the direction of applied temperature difference, is calculated from converged temperature gradients at either of the hot and cold boundaries. The numerical model, used for this simulation, solves the 3D steady-state heat conduction equation using the finite volume method on Cartesian coordinates, where the detailed structure of the foam, obtained from 3D CT-scan images, is represented by the staircase approximation. The method is described in detail by Mendes et al. [6] and hence it is not presented here for the sake of brevity.

The simplified model for ETC, in the dimensionless form, used by Mendes et al. [6] is given as:

$$\tilde{k}_{\text{eff}} = b\tilde{k}_{\text{min}} + (1-b)\tilde{k}_{\text{max}} \quad (3)$$

where all thermal conductivities are made dimensionless with respect to the thermal conductivity of solid phase $k_s$, i.e., $\tilde{k} = k / k_s$. Further, $\tilde{k}_{\text{min}}$ and $\tilde{k}_{\text{max}}$ are the dimensionless generic minimum and maximum bounds for $\tilde{k}_{\text{eff}}$. The previous study of Mendes et al. [6] showed that the Hashin-Shtrikman bounds generally serve the best in this respect for all the investigated open-cell porous foams and are given as [11]:

$$\tilde{k}_{\text{min}} = \frac{k_f \left[ 2k_f + 1 - 2(k_f - 1)(1 - \phi_M) \right]}{2k_f + 1 + [k_f - 1](1 - \phi_M)} \quad (4a)$$

$$\tilde{k}_{\text{max}} = \frac{2 + k_f - 2(1 - k_f)\phi_M}{2 + k_f + (1 - k_f)\phi_M} \quad (4b)$$

It must be noted here that both lower and upper bounds of the dimensionless ETC can be uniquely determined from Eq. (4) using known values of $k_f = k_f / k_s$ and $\phi_M$ in a straightforward manner. Under vacuum condition, i.e. when $\tilde{k}_f = 0$, $\tilde{k}_{\text{min}} = 0$ and $\tilde{k}_{\text{max}} = 2(1 - \phi_M) / (2 + \phi_M)$ are obtained from Eq. (4). Thus, the adjustable parameter $b$ in Eq. (3) can be evaluated from the following expression [6]:

$$b = 1 - \frac{\tilde{k}_{\text{eff,s}}(2 + \phi_M)}{2(1 - \phi_M)} \quad (5)$$

where $\tilde{k}_{\text{eff,s}}$ is the dimensionless ETC of the foam under vacuum condition and is determined from the detailed numerical prediction of ETC only due to the presence of solid matrix by completely ignoring the contribution of fluid phase. This condition is ensured by applying the adiabatic boundary condition on solid-fluid interfaces and hence, calculations are omitted in the fluid region, which significantly reduces the requirement of computational time.

It is worth mentioning here that $\tilde{k}_{\text{eff,s}}$ implicitly contains the morphological information of porous foams in a quantitative manner that is relevant for heat transfer due to pure conduction. Additionally, its calculation is performed only once for a particular porous medium since the dimensionless result is independent of the
thermal conductivity of solid phase. To that extent, \(\tilde{k}_{\text{eff},s}\) as well as the parameter \(b\), in Eq. (5), can be recognized as geometric factors those solely depend on the internal structure of porous foams, as demonstrated by Mendes et al. [9].

3 Results and discussions

Effective thermal conductivities for two pure alumina foam samples, with 10 and 45 ppi (see Figure 1), are predicted at room temperature by considering the influence of both solid and fluid phases as well as by neglecting the contribution of fluid phase (vacuum condition). The fluid phase is assumed to be air (\(k_f = 0.026\) W/mK). Since the solid phase contains microscopic porosity, which could not be captured by the CT-scan information, \(k_s\) is smaller than the conductivity of pure alumina and is calculated here using the upper Hashin-Shtrikman bound, similar to Eq. (4b), given as:

\[
k_s = k_A \left( \frac{2k_A + k_f - 2(k_A - k_f)\phi_m}{2k_A + k_f + (k_A - k_f)\phi_m} \right)
\]

where \(\phi_m\), \(k_f\) and the thermal conductivity of pure alumina (\(k_A = 33\) W/mK [10]) are used as inputs.

The grid size used for discretizing the geometry of both foam samples is 750×750×750 based on the CT voxel resolution and the length of representative cubic samples. Therefore, in order to speed-up the computation of ETC, all numerical simulations are executed using parallel computation with 216 processors.

Table 1: Numerical prediction of dimensionless ETC under vacuum condition in different directions, along with the thermal conductivity ratio of solid phase to pure alumina, as functions of microscopic porosity.

<table>
<thead>
<tr>
<th>(\phi_m)</th>
<th>(k_s/k_A)</th>
<th>(\tilde{k}<em>{\text{eff},s} = k</em>{\text{eff},s}/k_s) (×10^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10 ppi foam</td>
<td>45 ppi foam</td>
</tr>
<tr>
<td>x</td>
<td>y</td>
<td>z</td>
</tr>
<tr>
<td>x</td>
<td>y</td>
<td>z</td>
</tr>
<tr>
<td>0</td>
<td>1.00</td>
<td>4.23</td>
</tr>
<tr>
<td>0.05</td>
<td>0.93</td>
<td>4.59</td>
</tr>
<tr>
<td>0.1</td>
<td>0.86</td>
<td>5.02</td>
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<tr>
<td>0.15</td>
<td>0.79</td>
<td>5.52</td>
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<tr>
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<td>0.73</td>
<td>6.03</td>
</tr>
<tr>
<td>0.25</td>
<td>0.67</td>
<td>6.65</td>
</tr>
</tbody>
</table>

Table 1 presents the predicted directional \(\tilde{k}_{\text{eff},s}\) as well as \(k_s/k_A\) as functions of microscopic porosity for both foam samples, which is required by the simplified model for ETC. It can be observed from the data that \(\tilde{k}_{\text{eff},s}\) is different in different directions, showing that both foam samples contain certain degree of anisotropy. Moreover, \(\tilde{k}_{\text{eff},s}\) always increases with the increase in \(\phi_m\), which is expected since according to Eq. (1), higher values of \(\phi_m\) yield lower values of \(\phi_M\) and consequently, a higher solid fraction including microscopic porosity is available for heat conduction. Table 1 also shows that as \(\phi_m\) increases, \(k_s\) decreases considerably comparatively to the thermal conductivity of pure alumina.

Figure 3: Comparison between detailed predictions and estimations from simplified model for directional ETC as functions of microscopic porosity for open-cell alumina foams: (top) 10 ppi; (bottom) 45 ppi.

Figure 3 presents the comparison between detailed predictions of the directional ETC and respective estimations from the simplified model as functions of microscopic porosity for both foam samples. It can be observed that \(k_{\text{eff}}\) increases with \(\phi_m\), which is in agreement with the respective behavior of \(\tilde{k}_{\text{eff},s}\) presented in Table 1. Nevertheless, it is important to highlight that this similar behavior of \(k_{\text{eff}}\) and \(\tilde{k}_{\text{eff},s}\) with \(\phi_m\) may not always occur because as \(\phi_m\) increases, the value of \(k_s\) decreases due to the influence of microscopic
pores in the bulk solid material. Therefore, depending on the foam structure and the thermal conductivity of bulk solid material, situations may occur where \( k_{\text{eff}} \) would decrease with \( \phi_m \).

Figure 4: Relative errors for estimations of the directional ETC obtained with the simplified model as functions of microscopic porosity for different open-cell alumina foams.

It can also be observed from Figure 3 that generally, estimated values of \( k_{\text{eff}} \) obtained from the simplified model, are in reasonably good agreement with results of detailed simulations, especially for the foam with larger pore size (10 ppi). Respective relative errors for estimations of the ETC are presented in Figure 4. One can easily observe from the figure that for the 10 ppi foam, the simplified model always yields relative errors lower that 1.5% whereas for the 45 ppi foam, the relative errors can go up to 11%, depending upon \( \phi_m \). The higher relative errors obtained for the foam with smaller pore size (45 ppi) are in agreement with the findings reported by Mendes et al. [6] and can be explained by the fact that although the pore size should not have any direct influence on the ETC [6], during the manufacturing process of foams, smaller pore sizes are expected to influence the distribution of material between lumps and struts of the structure by allowing more solid material being concentrated on lumps. As demonstrated by Mendes et al. [6], this odd distribution of material in the foam structure decreases the ETC and degrades the prediction of simplified model, as can be confirmed by the comparison between results for 10 and 45 ppi foams, shown in Figures 3 and 4. Nevertheless, a closer look into the results presented in Figure 3 reveals that even for the 45 ppi foam, since the maximum error is around 11%, the accuracy of simplified model can be considered sufficient in order to allow for a reliable parametric study on the ETC, with respect to both the degree of anisotropy of foams and the microscopic porosity.

Figure 5: Ratio of computation times for \( k_{\text{eff,s}} \) to ETC in different directions as functions of microscopic porosity for different open-cell alumina foams.

Figure 5 shows the ratio of computation time required for the evaluation of \( k_{\text{eff,s}} \) to that for the detailed simulation of ETC in different directions as functions of microscopic porosity for both foam samples. It can be observed from the figure that the ratio of computation time is hardly affected by both \( \phi_m \) and the direction of heat transfer and its value is generally around 5%. This result quantitatively exhibits the advantage of simplified modeling approach in terms of computational resources as compared to the detailed simulation of heat conduction for the estimation of ETC. This indicates that for the same computation time, the computational power can be reduced approximately 20 times or vice-versa.

**CONCLUSIONS**

In the present study, the ETC of open-cell ceramic foams is numerically predicted using both detailed heat conduction model and simplified approach proposed by Mendes et al. [6]. Two different pure alumina foam samples, with 10 and 45 ppi, are considered and foam geometries are obtained from 3D CT-scan images. The influence of both microscopic porosity within the bulk solid material and degree of anisotropy of foam structures on the ETC is investigated. Based on the comparison of results obtained from detailed and simplified models the following conclusions can be drawn:

1. The accuracy of simplified model can be considered sufficient enough in order to allow for a reliable parametric study on the ETC, regarding both the degree of anisotropy of foams and the influence of microscopic porosity.
2. The simplified approach requires significantly reduced computational time, which scales at least to the order of porosity of foams.
Therefore, the present investigation demonstrates that the sensitivity analysis of ETC to different parameters can be reliably accomplished with reasonable accuracy and reduced computational effort by using the simplified modeling approach of Mendes et al. [6], while relying upon the real morphology of open-cell porous foams.

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