7-4-2016

Stokes flow past a two-layers heterogeneous porous sphere with the effect of stress jump condition: An exact solution

Kun Yang
Huazhong University of Science and Technology, yangk@hust.edu.cn

Xin Yan
Huazhong University of Science and Technology

Kambiz Vafai
University of California, Riverside

Proceedings of the 6th International Conference on Porous Media and Its Applications in Science and Engineering
ICPM6
July 3-8, 2016, Waikoloa, Hawaii

STOKES FLOW PAST A TWO-LAYER HETEROGENEOUS POROUS SPHERE WITH THE EFFECT OF STRESS JUMP CONDITION: AN EXACT SOLUTION

Kun Yang*, Xin Yan
Huazhong University of Science and Technology, Wuhan, Hubei, 430074, China

Kambiz Vafai*
Department of Mechanical Engineering, University of California, Riverside, Riverside, CA 92521-0425, USA

ABSTRACT
A heterogeneous porous sphere containing two-layer porous medium with internal radius r_1 and external radius r_2 is considered, which is immersed in a uniform stream of the inflow velocity U. The internal porous region, the external porous region and the free fluid region are denoted by regions I, II and III respectively. Darcy-Brinkman equations are adopted to describe the flow in region I and II, and Stokes equations are adopted to describe the flow in region III. The continuity of the velocity components and stresses are taken at the interface between region I and II. The continuity of the velocity components and normal stress and tangential stress jump conditions are taken at the interface between region II and III. The exact flow and drag solutions are derived and verified in some limiting cases by comparing with the solutions derived in other researchers' works. In addition, it is found that both the permeability and the stress jump coefficient have significant effect on the drag.

Keywords: Stokes flow; two-layer heterogeneous porous sphere; stress jump condition; Darcy-Brinkman equation

INTRODUCTION
The flow past a porous sphere has extensive industrial and engineering applications[1-3], such as the flow in porous beds, the flow of pulverized coals particulate during combustion, sedimentation of fine particulate suspensions, the filtration of solids from liquids etc.

Stokes flow past a homogeneous porous sphere has been studied by many researchers[4-11]. Padmavathi and Amaranath[4] studied a general non-axisymmetric Stokes flow of viscous fluid past a porous sphere using the Darcy model in the porous medium. Vainshtein[5] investigated creeping flow past a porous spheroid. The Stokes and Darcy equations were used to govern the fluid outside and inside the porous spheroid respectively. Higdon and Kojima[6] discussed Stokes flow past porous particles. They adopted Darcy-Brinkman model to describe the flow in porous sphere. Yu and Kaloni[7] constructed a Cartesian tensor solution of Darcy-Brinkman equation for the uniform flow past a porous sphere and calculated the drag force on the sphere. By proposing a representation of the velocity and the pressure fields in a general non-axisymmetric Stokes flow, Padmavathi et al.[8] calculated the drag and torque for the porous sphere. The continuity conditions of the velocity and stress components at the porous/clear fluid interface were used in the above-mentioned works. However, Ochoa-Tapia and Whitaker[12, 13] investigated the boundary conditions at the porous/clear fluid interface. Their works resulted in the stress jump boundary conditions which require a discontinuity in the tangential stress but continuity in the velocity components and normal stress. Srivastava[10] used the conditions suggested by Ochoa-Tapia and Whitaker[12, 13] to discuss flow past a porous sphere at low Reynolds number. Sekhar et al.[11] found that the stress jump coefficient had a great effect on the drag of the porous sphere.

The composite structure of a porous sphere with an impermeable core is an expansion of a completely porous sphere. Sekhar and Amaranath[14] used Darcy's law in the porous region and Stokes equations in the fluid region for Stokes flow past this structure. Padmavathi and Amaranath[15] proposed a solution of Darcy-Brinkman equation for an arbitrary Stokes flow. Bhattacharyya and Raja[16, 17] considered the stress jump boundary conditions at the interface between the clear fluid and porous region to calculate the drag force and torque. The uniform Stokes flow past a porous sphere with a rigid core with the stress jump boundary conditions was discussed by Srivastava[18].

Another expansion structure is a porous sphere containing concentric a spherical cavity. Bhatt and Sacheti[19] derived the exact solution for the flow past a porous spherical shell and discussed the drag on the sphere for different values of governing parameters by adopting the Darcy-Brinkman model in the porous shell. Hsu et al.[20] investigated the advantages by adopting Darcy-Brinkman model in the porous shell. Keh and Lu[21] researched the

*Corresponding author, email: yangk@hust.edu.cn(Kun Yang), vafai@engr.ucr.edu (Kambiz Vafai)

impacts of governing parameters on the motilities for a porous spherical shell by solving the Stokes and Darcy-Brinkman equations respectively.

Only one-layer homogeneous porous medium was considered for above-mentioned cases. In this paper, a heterogeneous porous sphere containing two-layer porous media with internal radius r_1 and external radius r_2 is considered, which is immersed in a uniform flow with velocity U. As boundary conditions, continuity of the velocity components, and the stress jump condition at the porous/fluid interface, continuity of the velocity and stress components at the porous/porous interface are adopted. The exact solutions of velocity distribution and drag force on the surface of the sphere are determined. Moreover, these solutions are verified in some limiting cases.

1 Mathematical formulation

The problem is concerned by dividing the flow in three regions (Fig.1): I is the region of the internal porous sphere, II is the region of the external porous sphere, III is the region of the clear fluid.

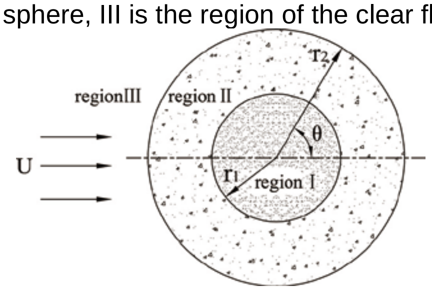

Fig.1: The physical model of the problem

The flow in region I $(0 \le \tilde{r} \le r_1)$ and II $(r_1 \le \tilde{r} \le r_2)$ is governed by the Darcy-Brinkman model:

$$\tilde{\nabla}\tilde{P}_i = -\frac{\mu}{k_i}\tilde{V}_i + \mu_{ei}\tilde{\nabla}^2\tilde{V}_i \tag{1}$$

$$\tilde{\nabla}\cdot\tilde{V}_i = 0 \tag{2}$$

And the flow in region III ($\tilde{r} \ge r_2$) is governed by the Stokes equations:

$$\tilde{\nabla}\tilde{P}_3 = \mu\tilde{\nabla}^2\tilde{V}_3 \tag{3}$$

$$\tilde{\nabla}\cdot\tilde{V}_3 = 0 \tag{4}$$

where $\tilde{V}_i, \tilde{P}_i, \mu_{ei}$ and k_i , i=1, 2, are the velocity vector, pressure, effective viscosity and permeability of the internal and external porous medium respectively. $\tilde{V}_3$, $\tilde{P}_3$ and μ are the velocity vector, pressure and viscosity of the clear fluid flow.

Considering the axial symmetry of the motion, we take $(\tilde{u}_{ri}, \tilde{u}_{\theta i}, 0)$, i=1,2,3, as the velocity components in the directions of $(\tilde{r}, \theta, \phi)$ in the regions I, II, III respectively. The Stokes' stream function $\tilde{\varphi}_i$, i=1,2, 3, in the spherical polar coordinates is defined to satisfy the continuity Eqs.(2) and (4) by:

$$\tilde{u}_{ri} = \frac{1}{(\tilde{r})^2\sin\theta}\frac{\partial\tilde{\varphi}_i}{\partial\theta}, \quad \tilde{u}_{\theta i} = -\frac{1}{\tilde{r}\sin\theta}\frac{\partial\tilde{\varphi}_i}{\partial\tilde{r}} \tag{5}$$

2 Method of solution

2.1 Dimensionless governing equations

The flowing dimensionless variables are introduced:

$$\gamma_i^2 = \frac{\mu_{ei}}{\mu}, \sigma_i^2 = \frac{r_2^2}{k_i}, \alpha_i^2 = \frac{\sigma_i^2}{\gamma_i^2}, i = 1,2$$

$$r = \frac{\tilde{r}}{r_2}, \eta = \cos\theta, \lambda = \frac{r_1}{r_2}, R_e = \frac{Ur_2}{\nu}, \tilde{V} = V\cdot U \tag{6}$$

$$\nabla = \tilde{\nabla}\cdot r_2, \nabla^2 = \tilde{\nabla}^2\cdot r_2^2, \varphi_i = \frac{\tilde{\varphi}_i}{r_2^2 U}, P_i = \frac{r_2}{\mu U}\tilde{P}_i, i = 1,2,3$$

The governing Eqs.(1) and (3) for region I, II, III can be rewritten as:

$$D^4\varphi_i - \alpha_i^2 D^2\varphi_i = 0 \ (i = 1, 2) \tag{7}$$

$$D^4\varphi_3 = 0 \tag{8}$$

where D is defined as:

$$D^2 = \frac{\partial^2}{\partial r^2} + \frac{1-\eta^2}{r^2}\frac{\partial^2}{\partial\eta^2} \tag{9}$$

2.2 Boundary conditions

At the interface between the region I and II, $r = \lambda$, we adopt the continuity conditions of the velocity and stress components:

$$\begin{gathered} u_{r1} = u_{r2}, u_{\theta 1} = u_{\theta 2} \\ \tau_{rr1} = \tau_{rr2}, \tau_{r\theta 1} = \tau_{r\theta 2} \end{gathered} \tag{10}$$

At the interface between the region II and III, $r = 1$, we use the continuity of the velocity components, continuity of the normal stress, and the stress jump condition given by Ochoa-Tapia and Whitaker[12, 13] for the tangential stress:

$$\begin{gathered} u_{r2} = u_{r3}, u_{\theta 2} = u_{\theta 3} \\ \tau_{rr2} = \tau_{rr3}, \tau_{r\theta 2} - \tau_{r\theta 3} = \beta\sigma_2 u_{\theta 3} \end{gathered} \tag{11}$$

where β is the stress jump coefficient.

$$u_{r3} = \cos\theta, u_{\theta 3} = -\sin\theta \text{, as } r \to \infty \tag{12}$$

$$u_{r1}, u_{\theta 1} \text{ should be finite, as } r \to 0 \tag{13}$$

2.3 Solutions

The Eq.(7) gives(for i=1, 2):

$$\varphi_i = \frac{(1-\eta^2)}{r}\begin{bmatrix} K_i r^3 + N_i(\sinh\alpha_i r - \alpha_i r\cosh\alpha_i r) \\ +M_i + G_i(\cosh\alpha_i r - \alpha_i r\sinh\alpha_i r) \end{bmatrix} \tag{14}$$

The velocity distributions are given by(for i=1, 2):

$$u_{ri} = \frac{2\eta}{r^3}\begin{bmatrix} K_i r^3 + N_i(\sinh\alpha_i r - \alpha_i r\cosh\alpha_i r) \\ +M_i + G_i(\cosh\alpha_i r - \alpha_i r\sinh\alpha_i r) \end{bmatrix} \tag{15}$$

$$u_{\theta i} = \frac{\sin\theta}{-r^3}\left\{\begin{matrix} 2K_i r^3 - M_i + \\ N_i[\alpha_i r\cosh\alpha_i r - (1+\alpha_i^2 r^2)\sinh\alpha_i r] \\ G_i[\alpha_i r\sinh\alpha_i r - (1+\alpha_i^2 r^2)\cosh\alpha_i r] \end{matrix}\right\} \tag{16}$$

The corresponding stresses are given by(for i=1, 2):

$$\tau_{r\theta i} = \gamma_i^2(\frac{1}{r}\frac{\partial u_{ri}}{\partial\theta} - \frac{u_{\theta i}}{r} + \frac{\partial u_{\theta i}}{\partial r})$$

$$= \frac{\gamma_i^2\sin\theta}{-r^4}\left\{\begin{matrix} 6M_i + N_i[(6+3\alpha_i^2 r^2)\sinh\alpha_i r - (6\alpha_i r \\ +\alpha_i^3 r^3)\cosh\alpha_i r] + G_i[(6+3\alpha_i^2 r^2) \\ \cosh\alpha_i r - (6\alpha_i r + \alpha_i^3 r^3)\sinh\alpha_i r] \end{matrix}\right\} \tag{17}$$

$$\tau_{rri} = -P_i + 2\gamma_i^2 \frac{\partial u_{ri}}{\partial r}$$

$$= \frac{\gamma_i^2 \eta}{-r^4} \left\{ \begin{array}{l} -2K_i \alpha_i^2 r^5 + \left(\alpha_i^2 r^2 + 12\right) M_i + \\ N_i [(12 + 4\alpha_i^2 r^2) \sinh \alpha_i r - 12\alpha_i r \cosh \alpha_i r] \\ G_i [(12 + 4\alpha_i^2 r^2) \cosh \alpha_i r - 12\alpha_i r \sinh \alpha_i r] \end{array} \right\} \quad (18)$$

While Eq.(8) gives:

$$\varphi_3 = (1-\eta^2)[Ar^{-1} + Br + Cr^2 + Dr^4] \quad (19)$$

The velocity distributions are given by:

$$u_{r3} = 2\eta\left(Ar^{-3} + Br^{-1} + C + Dr^2\right) \quad (20)$$

$$u_{\theta 3} = -\sin\theta\left(-Ar^{-3} + Br^{-1} + 2C + 4Dr^2\right) \quad (21)$$

The corresponding stresses can be obtained by:

$$\tau_{r\theta 3} = \frac{1}{r}\frac{\partial u_{r3}}{\partial \theta} - \frac{u_{\theta 3}}{r} + \frac{\partial u_{\theta 3}}{\partial r} = -6\sin\theta\left(Ar^{-4} + Dr\right) \quad (22)$$

$$\tau_{rr3} = -P_3 + 2\frac{\partial u_3}{\partial r} = -6\eta\left(2Ar^{-4} + Br^{-2} + 2Dr\right) \quad (23)$$

where the constants $A, B, C, D, K_i, M_i, N_i, G_i, i = 1, 2$ can be determined by using the boundary conditions:

$$A = \Delta \begin{bmatrix} (s_{16}s_{20} - s_{17}s_{19})(s_1s_4 - s_2s_3) + (s_{14}s_{19} - s_{13}s_{20}) \\ (s_1s_6 - s_2s_5) + (s_{13}s_{17} - s_{14}s_{16})(s_1s_8 - s_2s_7) + \\ (s_{10}s_{20} - s_{11}s_{19})(s_3s_6 - s_4s_5) + (s_{11}s_{16} - s_{10}s_{17}) \\ (s_3s_8 - s_4s_7) + (s_{10}s_{14} - s_{11}s_{13})(s_5s_8 - s_6s_7) \end{bmatrix} \quad (24)$$

$$B = \Delta \begin{bmatrix} (s_{17}s_{18} - s_{15}s_{20})(s_1s_4 - s_2s_3) + (s_{12}s_{20} - s_{14}s_{18}) \\ (s_1s_6 - s_2s_5) + (s_{14}s_{15} - s_{12}s_{17})(s_1s_8 - s_2s_7) + \\ (s_{11}s_{18} - s_9s_{20})(s_3s_6 - s_4s_5) + (s_{17}s_9 - s_{11}s_{15}) \\ (s_3s_8 - s_4s_7) + (s_{11}s_{12} - s_9s_{14})(s_5s_8 - s_6s_7) \end{bmatrix} \quad (25)$$

$$C = 1/2, D = 0 \quad (26)$$

$$K_1 = \frac{(s_3s_9 - s_1s_{12})A + (s_3s_{10} - s_1s_{13})B + s_3s_{11} - s_1s_{14}}{s_2s_3 - s_1s_4} \quad (27)$$

$$N_1 = \frac{(s_2s_{12} - s_4s_9)A + (s_2s_{13} - s_4s_{10})B + s_2s_{14} - s_4s_{11}}{s_2s_3 - s_1s_4} \quad (28)$$

$$M_1 = 0, G_1 = 0 \quad (29)$$

$$K_2 = s_1K_1 + s_2N_1 \quad (30)$$

$$M_2 = s_2K_1 + s_4N_1 \quad (31)$$

$$N_2 = s_5K_1 + s_6N_1 \quad (32)$$

$$G_2 = s_7K_1 + s_8N_1 \quad (33)$$

$$\Delta = \left\{ \begin{array}{l} s_9\left[s_7(s_4s_{16} - s_6s_{13}) + s_8(s_5s_{13} - s_3s_{16})\right] + \\ (s_{15}s_{19} - s_{16}s_{18})(s_1s_4 - s_2s_3) + (s_{13}s_{18} - s_{12}s_{19}) \\ (s_1s_6 - s_2s_5) + (s_{12}s_{16} - s_{13}s_{15})(s_1s_8 - s_2s_7) \\ +(s_9s_{19} - s_{10}s_{18})(s_3s_6 - s_4s_5) + \\ s_{10}\left[s_{12}(s_6s_7 - s_5s_8) + s_{15}(s_3s_8 - s_4s_7)\right] \end{array} \right\}^{-1} \quad (34)$$

with:

$$s_1 = (2\sigma_1^2 + \sigma_2^2)/(3\alpha_2^2\gamma_2^2) \quad (35)$$

$$s_2 = \{[\lambda^2(\sigma_2^2 - \sigma_1^2) + 6(\gamma_2^2 - \gamma_1^2)(\alpha_1^2\lambda^2 + 3)]\sinh(\alpha_1\lambda) \\ +\alpha_1\lambda[\lambda^2(\sigma_1^2 - \sigma_2^2) + 18(\gamma_1^2 - \gamma_2^2)]\cosh(\alpha_1\lambda)\}/(3\alpha_2^2\lambda^5\gamma_2^2) \quad (36)$$

$$s_3 = 2\lambda^3(\sigma_2^2 - \sigma_1^2)/(3\alpha_2^2\gamma_2^2) \quad (37)$$

$$s_4 = 2(\sigma_1^2 - \sigma_2^2)[\alpha_1\lambda\cosh(\alpha_1\lambda) - \sinh(\alpha_1\lambda)]/(3\alpha_2^2\gamma_2^2) \quad (38)$$

$$s_5 = 2(\sigma_1^2 - \sigma_2^2)[\cosh(\alpha_2\lambda) - \alpha_2\lambda\sinh(\alpha_2\lambda)]/(\alpha_2^5\gamma_2^2) \quad (39)$$

$$s_{6-1} = \{\lambda^2(\sigma_2^2 - \sigma_1^2) + 3(\gamma_2^2 - \gamma_1^2)[2\lambda^2(\alpha_1^2 + \alpha_2^2) \\ +\alpha_1^2\alpha_2^2\lambda^4 + 6]\}\sinh(\alpha_1\lambda) + \alpha_1\lambda[\lambda^2(\sigma_1^2 - \sigma_2^2) \\ +6(\gamma_1^2 - \gamma_2^2)(\alpha_2^2\lambda^2 + 3) + \alpha_2^2\sigma_1^2\lambda^4]\cosh(\alpha_1\lambda) \quad (40)$$

$$s_{6-2} = \alpha_2\lambda[\lambda^2(\sigma_1^2 - \sigma_2^2) + 6(\gamma_1^2 - \gamma_2^2)(\alpha_1^2\lambda^2 + 3) \\ -\alpha_1^2\sigma_2^2\lambda^4]\sinh(\alpha_1\lambda) + \alpha_1\alpha_2\lambda^2[\lambda^2(\sigma_2^2 - \sigma_1^2) + \\ 18(\gamma_2^2 - \gamma_1^2)]\cosh(\alpha_1\lambda) \quad (41)$$

$$s_6 = [s_{6-1}\cosh(\alpha_2\lambda) + s_{6-2}\sinh(\alpha_2\lambda)]/(\alpha_2^5\lambda^5\gamma_2^2) \quad (42)$$

$$s_7 = 2(\sigma_2^2 - \sigma_1^2)\left[\sinh(\alpha_2\lambda) - \alpha_2\lambda\cosh(\alpha_2\lambda)\right]/(\alpha_2^5\gamma_2^2) \quad (43)$$

$$s_8 = -\left[s_{6-1}\sinh(\alpha_2\lambda) + s_{6-2}\cosh(\alpha_2\lambda)\right]/(\alpha_2^5\lambda^5\gamma_2^2) \quad (44)$$

$$s_9 = (\sigma_2^2 + \beta\sigma_2 + 18\gamma_2^2 - 18)/(3\alpha_2^2\gamma_2^2) \quad (45)$$

$$s_{10} = (\sigma_2^2 - \beta\sigma_2 + 6\gamma_2^2 - 6)/(3\alpha_2^2\gamma_2^2) \quad (46)$$

$$s_{11} = (\sigma_2 - 2\beta)/(6\alpha_2\gamma_2) \quad (47)$$

$$s_{12} = 2(\sigma_2 + \beta)/(3\alpha_2\gamma_2) \quad (48)$$

$$s_{13} = 2\left[\sigma_2(\sigma_2 - \beta) + 3\right]/(3\alpha_2^2\gamma_2^2) \quad (49)$$

$$s_{14} = (\sigma_2 - 2\beta)/(3\alpha_2\gamma_2) \quad (50)$$

$$s_{15} = \{[\sigma_2\beta(1+\alpha_2^2) + 6(\gamma_2^2 - 1)(\alpha_2^2 + 3) + \sigma_2^2]\cosh(\alpha_2) \\ +[\alpha_2(18 - 18\gamma_2^2 - \sigma_2^2) - \beta\alpha_2\sigma_2]\sinh(\alpha_2)\}/(\alpha_2^5\gamma_2^2) \quad (51)$$

$$s_{16} = \{[6(\gamma_2^2 - 1) - \beta\sigma_2(1+\alpha_2^2) + \sigma_2^2]\cosh(\alpha_2) \\ +\beta\alpha_2\sigma_2 + \alpha_2(\sigma_2^2 - 6\gamma_2^2 + 6)\sinh(\alpha_2)\}/(\alpha_2^5\gamma_2^2) \quad (52)$$

$$s_{17} = \{-[\beta(1+\alpha_2^2) + \sigma_2]\cosh(\alpha_2) \\ +\alpha_2(\beta + \sigma_2)\sinh(\alpha_2)\}/(\alpha_2^4\gamma_2) \quad (53)$$

$$s_{18} = -\{[\sigma_2\beta(1+\alpha_2^2) + 6(\gamma_2^2 - 1)(\alpha_2^2 + 3) + \sigma_2^2]\sinh(\alpha_2) \\ +[\alpha_2(18 - 18\gamma_2^2 - \sigma_2^2) - \beta\alpha_2\sigma_2]\cosh(\alpha_2)\}/(\alpha_2^5\gamma_2^2) \quad (54)$$

$$s_{19} = \{[6(\gamma_2^2 - 1) - \beta\sigma_2(1+\alpha_2^2) + \sigma_2^2]\sinh(\alpha_2) \\ +[\beta\alpha_2\sigma_2 + \alpha_2(\sigma_2^2 - 6\gamma_2^2 + 6)]\cosh(\alpha_2)\}/(-\alpha_2^5\gamma_2^2) \quad (55)$$

$$s_{20} = \{-[\beta(1+\alpha_2^2) + \sigma_2]\sinh(\alpha_2) \\ +\alpha_2(\beta + \sigma_2)\cosh(\alpha_2)\}/(\alpha_2^4\gamma_2) \quad (56)$$

The drag on the surface of the sphere is given by:

$$\tilde{D}r = \iint_A (\tilde{\tau}_{\tilde{r}\tilde{r}3}\Big|_{\tilde{r}=r_2}\cos\theta - \tilde{\tau}_{\tilde{r}\theta 3}\Big|_{\tilde{r}=r_2}\sin\theta)dA = -8\pi\mu UBr_2 \quad (57)$$

When the sphere is impermeable, it can be derived that B=-3/4. And the drag is given by:

$$\tilde{D}r^* = 6\pi\mu Ur_2 \quad (58)$$

which is the Stokes' classical solution[22]. And the dimensionless drag can be defined as:

$$Dr = \tilde{D}r/(6\pi\mu Ur_2) = -4B/3 \quad (59)$$

3 Limiting cases for Stokes flow

The analytical solutions can be verified in some limiting cases.

3.1 Flow past a homogeneous porous sphere

When the internal and external porous medium have the same properties ($\gamma_1^2 = \gamma_2^2, \sigma_1 = \sigma_2$) or the internal radius r_1 approaches zero ($\lambda \to 0$), the two-layer heterogeneous porous sphere can be viewed as a homogeneous sphere. Some researchers[8, 9, 16, 17, 19] took $\gamma^2=1$, i.e. $\mu_e=\mu$, so that the calculations can be much simplified. Here by taking $\gamma_2^2=1$, the following expression for D_r is given by:

$$Dr = \frac{2\sigma_2^2[\sigma_2(\beta+\sigma_2)\cosh\sigma_2 - (\sigma_2^2\beta+\beta+\sigma_2)\sinh\sigma_2]}{\left\{\begin{matrix}\sigma_2(2\sigma_2^2+3)(\beta+\sigma_2)\cosh\sigma_2 - \\ [(2\sigma_2^4+3\sigma_2^2+3)\beta+3\sigma_2]\sinh\sigma_2\end{matrix}\right\}} \quad (60)$$

which agrees with the solution of Srivastava[10]. If we take the stress conditions to be continuous at the porous/clear fluid interface, which means $\beta=0$, we get the following expression for D_r:

$$Dr = \frac{2\sigma^2\left[\sigma\cosh\sigma - \sinh\sigma\right]}{\sigma(2\sigma^2+3)\cosh\sigma - 3\sinh\sigma} \quad (61)$$

which is identical with the dimensionless solution given by Yu and Kaloni[7] and Padmavathi and Amaranath[4].

3.2 Flow past a porous sphere with a solid core

When the permeability in internal region approaches zero ($k_1 \to 0$, i.e. $\sigma_1 \to \infty$), the porous sphere reduces to the porous spherical shell containing an impermeable core. Therefore, by taking limit of σ_1 in the expressions (25) and (59) tending to infinity, the value of the corresponding drag force can be found.

Table 1 gives the values of D_r for $\sigma_1 \to \infty$. Srivastava[18] calculated the values of the drag force for Stokes flow past a porous sphere with a rigid core. But the coefficient of the constant M in the Eq.(32) of their work[18] should be 12 instead of 24, and the coefficient of the constant A on the right side of the the Eq.(33) of their work[18] is supposed to be -1 instead of 1. It can be found in Table.1 that the values calculated in present work are exact the same as that calculated from the revised equations in the work of Srivastava[18].

Table 1. The values of D_r for various values of λ, σ_2 for $\sigma_1 \to \infty, \gamma_2^2 = 1, \beta = -0.5$

$\sigma_2 \setminus \lambda$		0.25	0.5
5	present work	0.83054	0.83929
	Srivastava's work	0.83054	0.83929
6	present work	0.86144	0.86637
	Srivastava's work	0.86144	0.86637
7	present work	0.88339	0.88642
	Srivastava's work	0.88339	0.88642

3.3 Flow past a porous spherical shell with a concentric spherical cavity

When the permeability in the internal region reaches infinity ($k_1 \to \infty$, i.e. $\sigma_1 \to 0$), the current structure turns out to be a porous spherical shell with a concentric spherical cavity. The solution can also be found by taking limit of σ_1 in the expressions (25) and (59) tending to 0. Bhatt and Sacheti[19] derived the exact solution of the drag with the continuity conditions of the velocity components, normal and tangential stresses at the porous/clear fluid interface for this case.

Since the expression is much lengthy, to verify our solution we have calculated some values of D_r for $\sigma_1 \to 0$. It can be concluded in Table.2 that the values calculated in present work and Bhatt's work are exactly the same.

Table 2. The values of D_r for various values of λ, σ_2 for $\sigma_1 \to 0, \gamma_2^2 = 1, \beta = 0$

$\sigma_2 \setminus \lambda$		0.25	0.5
5	present work	0.76298	0.75821
	Bhatt's work	0.76298	0.75821
6	present work	0.80512	0.80203
	Bhatt's work	0.80512	0.80203
7	present work	0.83505	0.83288
	Bhatt's work	0.83505	0.83288

4 Results and discussion

The drag force on the porous sphere D_r is a function of six pertinent parameters $(\gamma_1, \sigma_1, \gamma_2, \sigma_2, \lambda, \beta)$.

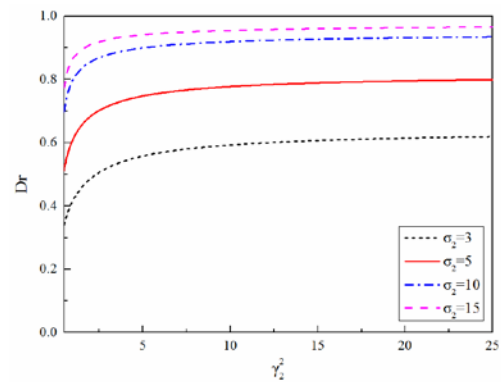

Fig.2: Variation of D_r with γ_2^2 for $\gamma_1 = 1, \sigma_1 = 5, \lambda = 0.5$, $\beta = 0.5$

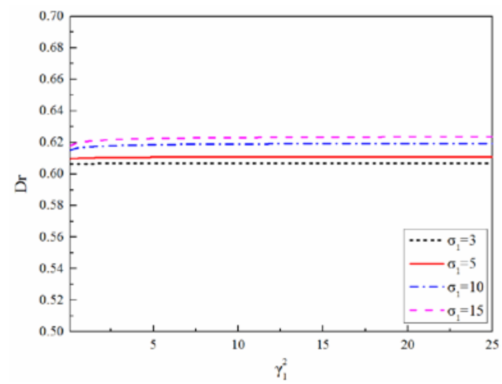

Fig.3: Variation of D_r with γ_1^2 for $\gamma_2 = 1, \sigma_2 = 5, \lambda = 0.5$, $\beta = 0.5$

The dependence of the dimensionless drag D_r on the external viscosity ratio γ_2 is presented in Fig.2

for different values of σ_2. Fig.3 shows the variation of D_r with the internal viscosity ratio γ_1 for different values of σ_1. Fig.2 shows that, as the external viscosity ratio increases, there is a slight increase in the drag at small external viscosity ratio. However, Fig.3 shows that the drag is almost unaffected with the variation of the internal viscosity ratio.

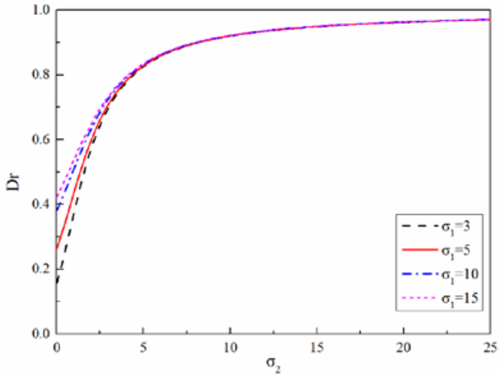

Fig.4: Variation of D_r with σ_2 for $\gamma_1 = 1, \gamma_2 = 1, \lambda = 0.5$, $\beta = -0.5$

Fig.4 shows the variation of D_r with σ_2 for different values of σ_1. It can be found that the drag increases significantly with the increase of σ_2. For small σ_2, the increasing of the drag with σ_2 is more obvious and the drag will increase with σ_1. With increasing in σ_2, the difference between the drags calculated from different σ_1 becomes small. For large value of σ_2, the drag will be independent of σ_1.

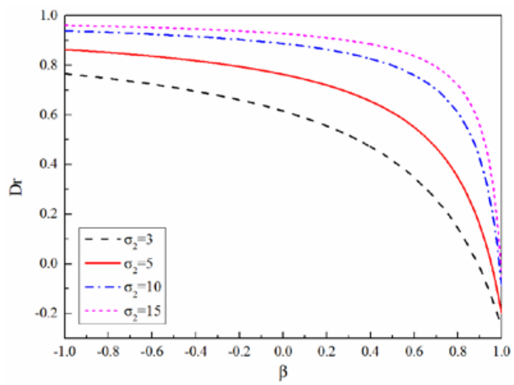

Fig.5: Variation of D_r with β for $\gamma_1 = 1, \gamma_2 = 1, \lambda = 0.5$, $\sigma_1 = 5$

Fig.5 shows the variation of D_r with the stress jump coefficient β for different values of σ_2. It can be concluded that the drag decreases with the increase of β, and it will decrease sharply for large β, which shows the influence of the stress jump condition at the porous /clear fluid interface cannot be ignored. In addition, it is interesting to find that the value of D_r becomes negative for some large values of β. If the drag is considered to be a positive value, there is a corresponding range of β which can be obtained based on the solutions in present work.

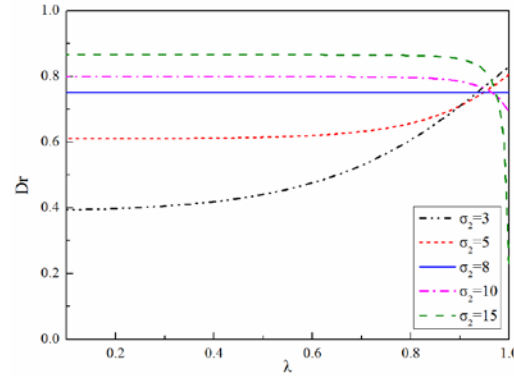

Fig.6: Variation of D_r with λ for $\gamma_1 = 1, \gamma_2 = 1, \beta = 0.5$, $\sigma_1 = 8$

The variation of D_r with λ for different σ_2 is drawn in Fig. 6. It can be found that, when $\sigma_2 < \sigma_1$, the drag increases as the inner to outer ratio increases since the average permeability of the whole region decreases as the inner to outer ratio increases. On the other hand, when $\sigma_2 > \sigma_1$, the drag decreases as the inner to outer ratio increases. When $\sigma_2 = \sigma_1$, the drag remains constant.

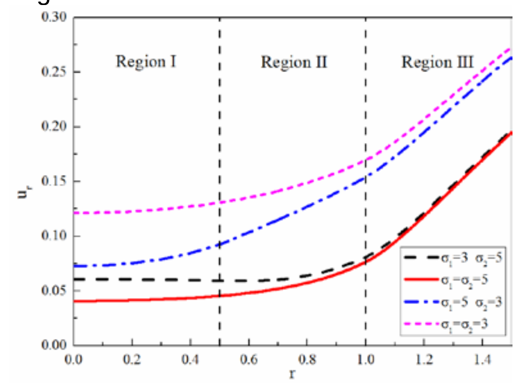

Fig.7: Variation of u_r with r for $\gamma_1 = 1, \gamma_2 = 1, \beta = -0.5$, $\theta = \pi / 4$

Fig.7 presents the variation of the radial velocity u_r for different values of σ_2 and σ_1 at $\theta = \pi / 4$. Fig.7 shows that the radial velocity of fluid for the case when σ_2 and σ_1 are small is greater than that for the case when σ_2 and σ_1 are large. Furthermore, the value of σ_2 has a remarkable influence on the velocity distributions in the region of the clear fluid.

CONCLUSIONS

From the above analysis, we can get the following conclusions:

(1) The exact flow and drag solutions for Stokes flow past a two-layer heterogeneous porous sphere with the stress jump condition are derived in this paper, which are equivalent to the exiting solutions in some limiting cases.

(2) Both the permeability and the stress jump coefficient have significant effects on the drag. The drag increases as the average permeability of the whole porous region decreases, and decreases with increase of the stress jump coefficient. In addition, the drag on the porous is more affected by the parameters of the external porous medium.
(3) The drag becomes negative for some large values of β. To keep a positive value of drag, the proper range of β can be obtained based on the solutions in this paper.

ACKNOWLEDGEMENT

This work is supported by the National Natural Science Foundation of China (No. 51476063).

REFERENCES

[1] Prakash J, Raja Sekhar G P, Kohr M(2011) Stokes flow of an assemblage of porous particles: stress jump condition. Zeitschrift für angewandte Mathematik und Physik. 62 (6): 1027-1046.

[2] Srivastava B G, Yadav P K, Deo S, et al(2014) Hydrodynamic permeability of a membrane composed of porous spherical particles in the presence of uniform magnetic field. Colloid Journal. 76 (6): 725-738.

[3] Yadav P K, Deo S, Yadav M K, et al(2013)On hydrodynamic permeability of a membrane built up by porous deformed spheroidal particles. Colloid Journal. 75 (5): 611-622.

[4] Padmavathi B, Amaranath T(1993)A solution for the problem of Stokes flow past a porous sphere. Zeitschrift für angewandte Mathematik und Physik ZAMP. 44 (1): 178-184.

[5] Vainshtein P, Shapiro M, Gutfinger C(2002) Creeping flow past and within a permeable spheroid. International journal of multiphase flow. 28 (12): 1945-1963.

[6] Higdon J, Kojima M(1981)On the calculation of Stokes' flow past porous particles. International Journal of Multiphase Flow. 7 (6): 719-727.

[7] Yu Q, Kaloni P(1988)A Cartesian tensor solution of the Brinkman equation. Journal of engineering mathematics. 22 (2): 177-188.

[8] Padmavathi B, Amaranath T, Nigam S(1993) Stokes flow past a porous sphere using Brinkman's model. Zeitschrift für angewandte Mathematik und Physik ZAMP. 44 (5): 929-939.

[9] Prakash J, Raja Sekhar G P(2011)Arbitrary oscillatory Stokes flow past a porous sphere using Brinkman model. Meccanica.47 (5): 1079-1095.

[10] Srivastava A C, Srivastava N(2005)Flow past a porous sphere at small Reynolds number. Zeitschrift für angewandte Mathematik und Physik. 56 (5): 821-835.

[11] Sekhar G R, Partha M, Murthy P(2006)Viscous flow past a spherical void in porous media: effect of stress jump boundary condition. Journal of Porous Media. 9 (8): 745-767.

[12] Ochoa-Tapia J A, Whitaker S(1995)Momentum transfer at the boundary between a porous medium and a homogeneous fluid—I. Theoretical development. International Journal of Heat and Mass Transfer. 38 (14): 2635-2646.

[13] Ochoa-Tapia J A, Whitaker S(1995)Momentum transfer at the boundary between a porous medium and a homogeneous fluid—II. Comparison with experiment. International Journal of Heat and Mass Transfer. 38 (14): 2647-2655.

[14] Sekhar G R, Amaranath T(2000)Stokes flow inside a porous spherical shell. Zeitschrift für angewandte Mathematik und Physik ZAMP. 51 (3): 481-490.

[15] Padmavathi B, Amaranath T(1996)Stokes flow past a composite porous spherical shell with a solid core. Archives of Mechanics. 48 (2): 311-323.

[16] Bhattacharyya A, Raja Sekhar G P(2004) Viscous flow past a porous sphere with an impermeable core : effect of stress jump condition. Chemical Engineering Science. 59 (21): 4481-4492.

[17] Bhattacharyya A, Raja Sekhar G P(2005)Stokes flow inside a porous spherical shell: Stress jump boundary condition. Zeitschrift für angewandte Mathematik und Physik. 56 (3): 475-496.

[18] Srivastava A C, Srivastava N(2006)Flow of a viscous fluid at small Reynolds number past a porous sphere with a solid core. Acta Mechanica. 186 (1-4): 161-172.

[19] Bhatt B, Sacheti N C(1994)Flow past a porous spherical shell using the Brinkman model. Journal of Physics D: Applied Physics. 27 (1): 37-41.

[20] Hsu H J, Huang L H, Hsieh P C(2004)A re-investigation of the low Reynolds number uniform flow past a porous spherical shell. International Journal for Numerical and Analytical Methods in Geomechanics. 28 (14): 1427-1439.

[21] Keh H J, Lu Y S(2005)Creeping motions of a porous spherical shell in a concentric spherical cavity. Journal of Fluids and Structures. 20 (5): 735-747.

[22] Chester W, Breach D, Proudman I(1969)On the flow past a sphere at low Reynolds number. Journal of Fluid Mechanics. 37 (04): 751-760.