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FLOW OF SHALE GAS IN TIGHT ROCKS USING A NON-LINEAR TRANSPORT MODEL WITH PRESSURE DEPENDENT MODEL PARAMETERS

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ABSTRACT

A nonlinear gas transport model is used for reservoir simulations of single-phase gas through homogeneous tight rocks. The transport model is an advection-diffusion partial differential equation for the pressure field, $p = p(x,t)$. The apparent convection velocity, $U = U(p, p_x)$ and the apparent diffusivity, $D = D(p)$, are highly non-linear functions of the pressure. The model parameters are fully pressure dependent, giving the model more realism than previous models. With given rock properties such as the intrinsic permeability, K, and porosity and tortuosity parameters, the system is solved for future pressure distributions over a period of time.

1 INTRODUCTION

With energy demand increasing and conventional hydrocarbon reserves depleting, shale gas recovery from unconventional sources has quickly developed in to major field in the energy sector. But the technology is expensive. Efforts to reduce costs include predicting future recovery yields through modelling of the shale gas transport through tight porous media and applying numerical simulation methods. A major obstacle though is that currently little is known about the transport processes in such tight porous media, and modeling the system realistically enough to be useful is a major challenge. In fact, relatively little field data is available even to characterize rock properties well. There is therefore an urgent need to develop modeling and simulation methods and it applications to shale gas recovery.

In the petroleum industry, predictions of future reservoir pressure is important because it determines the outflow rate of gas and hydrocarbons [4, 9, 12]. The aim here is to describe a new transport model, and apply it to obtain the pressure over a period of time in a shale gas reservoir.

2 TRANSPORT MODEL

Recently, Ali [1] and Ali & Malik [2, 3] have developed a nonlinear gas transport model for single-phase gas flow through homogeneous tight rocks with much greater realism than previous models. The transport model is in the form of an advection-diffusion partial differential equation for the pressure field, $p = p(x,t)$,

$$\frac{\partial p}{\partial t} + U\frac{\partial p}{\partial x} = D\frac{\partial^2 p}{\partial x^2}, \quad a \leq x \leq b, \ \ t > 0 \qquad (1)$$

The apparent convection velocity, $U = U(p, p_x)$ and the apparent diffusivity, $D = D(p)$, are highly non-linear functions of the pressure. The model incorporate various flow regimes (slip, surface diffusion, transition, continuum) based upon the Knudsen number, Kn, and also includes the Forchhiemers turbulence correction term. In application, the model parameters and associated compressibility factors are fully pressure dependent, giving the model more realism than previous models, see [1, 6].

In equation (1) we have [1],

$$D(p) = \frac{FK_a}{\mu \xi_t} \qquad (2)$$

$$U(p, p_x) = -\xi_3(p) D(p)\ \frac{\partial p}{\partial x} \qquad (3)$$

where

$$F = \left[1 + \frac{\rho}{\mu} K_a \beta |u|\right]^{-1}$$

NOMENCLATURE

p	=	The pressure
x	=	Spatial variable
t	=	Time
u	=	Velocity
K_a	=	Apparent permeability
f	=	Physical property

Greek Symbols

ρ	=	Gas density
μ	=	Dynamic Viscosity
β	=	Turbulence factor
ϕ	=	porosity
ξ_f	=	Compressibility coefficient of f

The transport model contains many physical parameters, such as porosity, tortuosity, permeability, etc. A key feature of the model is that all model properties, say f, are kept fully pressure dependent, $f = f(p)$. This leads to the appearance of compressibility coefficients, ξ_f, associated with each physical parameter, defined by

$$\xi_f(p) = \frac{1}{f}\frac{d(f(p))}{dp} \tag{4}$$

The compressibility coefficients can be either derived from known or assumed correlations between physical quantities in the system, or must be modeled from empirical data [1, 20].

$\xi_3(p)$ is a combination of other compressibility coefficients.

A second important feature in the model is that it incorporates important physical processes, such as slip flow, transition flow, continuum flow, adsorption and desorption from the rock material, and a turbulence correction term for high flow regions, [7, 8].

The model is also time-dependent.

This means that the model contains a high degree of realism, more than previous models [10, 11, 13, 14, 15, 16, 17].

3 NUMERICAL METHOD

The nonlinear transport system in equation (1) together with initial and boundary conditions must be solved numerically. Because it is a nonlinear advection-diffusion system, care needs to be taken when high gradients appear in the solution, which is possible when the local Peclet number becomes large.

It was found that an implicit finite volume staggered grid arrangement, Figures 1, 2 and 3, with the velocity defined on the grid boundaries, with a flux limiter (2nd order van Leer) adequately solved the system [1, 2, 18]. The discretized system produced a tri-diagonal system of nonlinear algebraic equations,

$$A(p)\,p = S(p) \tag{5}$$

where A is the coefficient matrix, S is the vector of source terms of the right hand side, and p is the pressure vector at all grid pints for which we are solving. Equation (5) has to be linearized before inverting, and then iterated to convergence before moving on to the next time step.

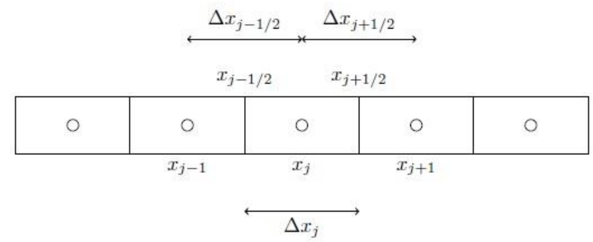

Figure 1: Control volume discretization of the 1-dimensional domain where the points x_j are chosen at the center of the block and the boundaries are located at the points $x_{j\pm 1/2}$.

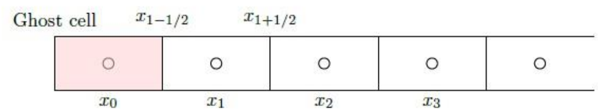

Figure 2: The left boundary condition is discretized by taking a ghost cell adjacent to the cell containing the point x_1.

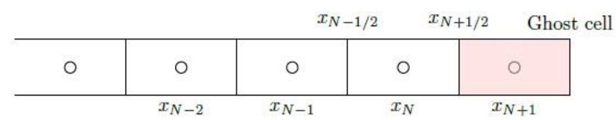

Figure 3: The right boundary condition is discretized by taking a ghost cell adjacent to the cell containing the point x_N.

The transient model with the turbulence correction term contains 15 parameters which are updated at every iteration step.

A sensitivity analysis on the parameters found that all parameters must be kept pressure dependent at all times for the most general application purposes [1, 2, 3].

The model was applied to determine the rock properties of shale core samples. Data was available from pressure pulse tests due to Pong [19]. The new model determined the porosity and the permeability to within realistic values for shale rocks, and with much better accuracy than previous models.

4 RESULTS

The new model, equation (1), was applied to simulate the pressure field in a shale rock core sample over a period of time, Figure 4.

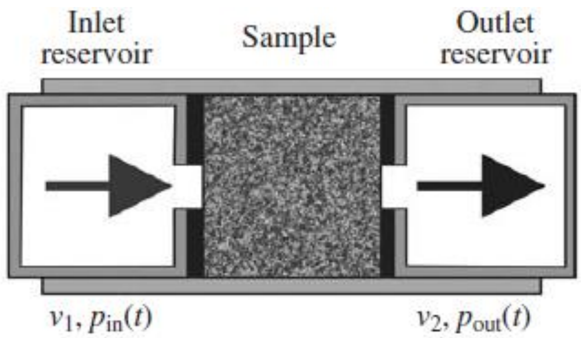

Figure 4: Transient pressure pulse test, Malkovsky [16].

Initial Condition

$$p(x,0) = \begin{cases} 100, & 0 < x \leq L, \\ 500, & x = 0 \end{cases} \qquad (6)$$

Boundary Conditions

$$p(0,t) = p_u(t), \qquad p(L,t) = p_d(t), \qquad (7)$$

Flux Conditions

$$\frac{\partial p}{\partial x} - \frac{\mu \varphi L \xi_\rho(p) V_u}{F K_a V_p} \frac{dp_u}{dt} = 0, \quad x = 0 \qquad (8)$$

$$\frac{\partial p}{\partial x} - \frac{\mu \varphi L \xi_\rho(p) V_d}{F K_a V_p} \frac{dp_d}{dt} = 0, \quad x = L, \qquad (9)$$

L is the length of the sample, V_u is the volume of the upstream reservoir, V_d is the volume of the downstream reservoir, V_p is the pore volume, p_u is the pressure in the upstream reservoir, p_d is the pressure in the downstream reservoir, F is the control factor.

We solve the new transient nonlinear transport model (1) with initial and flux conditions given in equations (6) – (9), to describe the pressure distribution in a rock core sample of length $L = 3\ mm$ to simulate pressure-pulse decay test. We obtain the pressure distribution under full pressure dependent reservoir parameters and compressibility coefficients.

We have carried out the numerical simulations using the data given in Table 7.6 of Ali [1]. A pressure pulse is induced in the upstream reservoir at $t = 0$, which is attached to a core plug containing a rock sample. Figures 5 – 9 show the results obtained from the numerical simulations.

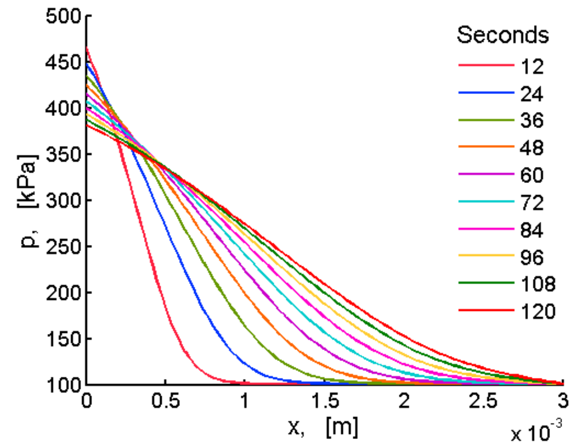

Figure 5: Numerical solutions from the new nonlinear transport model are plotted against distance x at short times (in seconds).

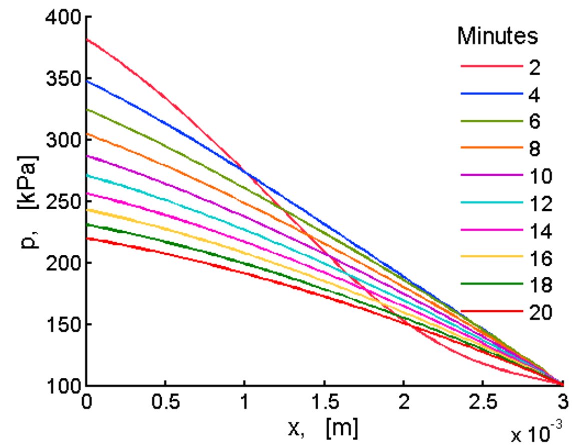

Figure 6: Numerical solutions from the new nonlinear transport model are plotted against distance x at long times (in minutes).

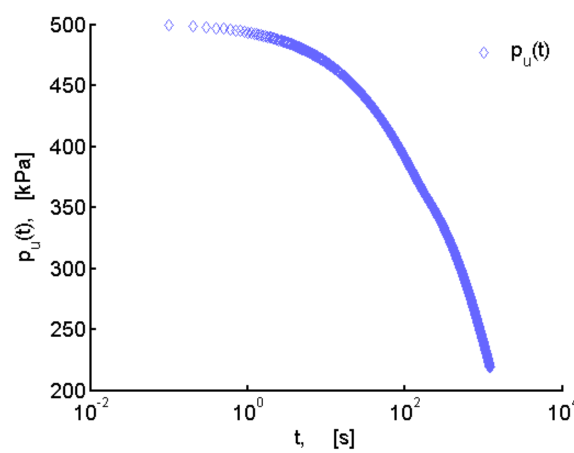

Figure 7: Pressure, $p_u(t)$, in the upstream reservoir is obtained from the new transport model and is plotted against time t. It shows an exponential decay of pressure with time.

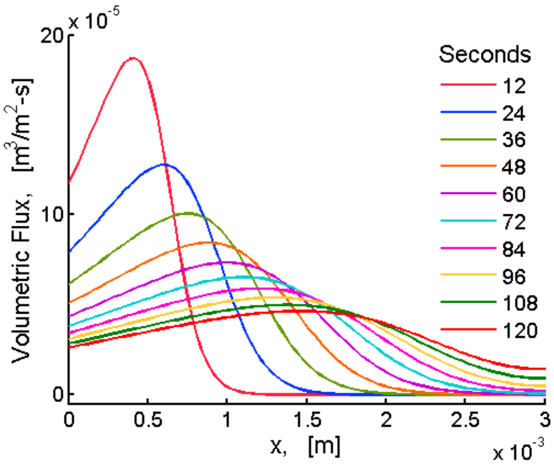

Figure 8: Volumetric flux u obtained from the new nonlinear transport model are plotted against distance x at short times (in seconds). We note that the volumetric flux greatly depends on the pressure gradient, it is very large for shorter times, say for $t < 2$ minutes.

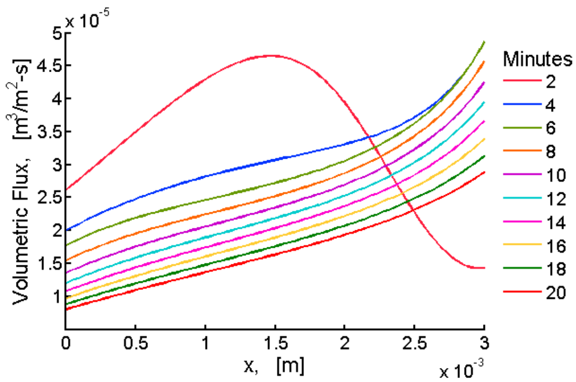

Figure 9: Plots of volumetric flux u against distance x. Here we have shown the long time behavior of u. We note that the magnitude of flux decreases with time but it decreases with the similar pattern. Moreover, the flux at the right boundary is greater than the flux at the inlet boundary.

5 CONCLUSIONS

The new nonlinear gas transport model for shale gas transport in tight porous media, equation (1), was used successfully to determine the pressure distribution in tight reservoirs over an extended period of time in a model rock core sample. Future work will concentrate upon embedding this model within a fractured rock structure.

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