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NATURAL CONVECTION IN HEAT EXCHANGERS
- A NEW INCENTIVE FOR MORE COMPACT HEAT EXCHANGERS ?

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ABSTRACT

Natural convection in the lab is very often studied at single surface elements or in single channels. The flow and heat transfer characteristics in these cases are more or less well understood and prediction of flow rates and heat transfer coefficients can usually be obtained from standard textbook formulae. While the behaviour of a single channel can safely be used to predict what happens in a bundle of parallel tubes in forced flow, the same is not true for natural convection. Starting from relatively simple cases of natural convection in single channels as well as in bundles of parallel ones, it is shown, that natural convection in bundles does behave completely differently. In the case of mixed convection in a vertical tube bundle, the effect of natural convection may lead to severe reductions in overall performance, but also—depending on the operation parameters—to an enhancement of heat transfer! Till now, the textbooks and handbooks on heat transfer do not even mention these possibilities, that may lead to a number of problems in heat exchanger operation practice.

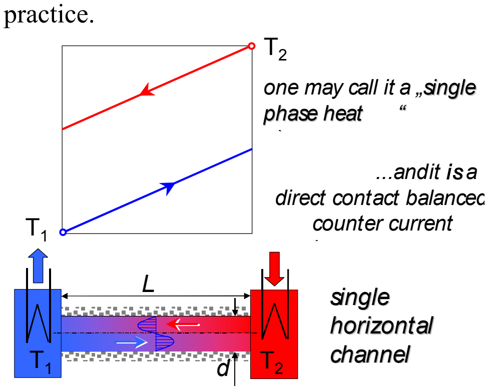

Fig. 1 Natural Convection in a Single Horizontal Channel.

WHAT IS THE PROBLEM?

The natural convective flow situation in a single horizontal channel is shown in Fig. 1. This arrangement may be called a “single phase heat pipe” as it does create a circulating flow between the cold side (T_1) and the hot side (T_2) with the hotter fluid on top of the colder one in two horizontal layers.

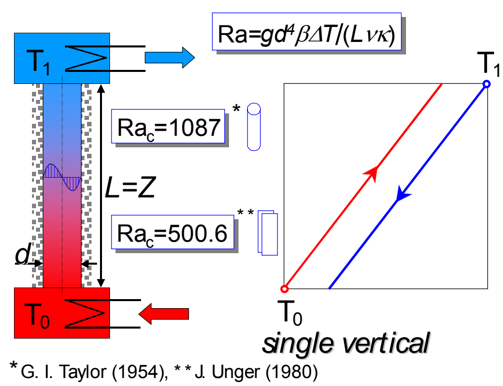

Fig. 2 Natural Convection in a Single Vertical Channel.

While the circulating flow in case of the horizontal channel starts as soon as there is a temperature difference greater zero, this is not true for the case of a vertical channel as shown in Fig. 2. Only if a critical value Ra_c of the (dimensionless) temperature difference Ra is surpassed, the circulating flow will start. For $Ra<Ra_c$ the fluid remains at rest. The critical Rayleigh numbers (temperature differences) have been calculated from a stability analysis for a cylindrical vertical tube by Taylor (1954) and for a vertical parallel plates duct by Unger (1980). The definition of Ra, as well as the values of Ra_c

from the literature are given in Fig. 2. Diameter (or gap width) d and length L enter the Rayleigh number as: d^4/L.

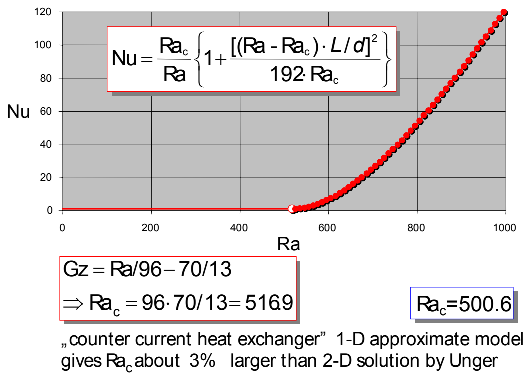

Fig. 3 Approximate solution to the natural convection problem in a single vertical parallel plates channel.

The single channel natural convection cases (Fig. 1 and 2) may be solved in a much simpler way (approximately) by a 1-D-model, taking into account their nature as direct contact balanced counter current heat exchangers. This is shown for the parallel plates duct in Fig. 3. The 1-D model in this case leads to simple explicit formulae for the flow rate (or Graetz number Gz) as a (linear) function of temperature difference (Rayleigh number) and predicts the critical Rayleigh number about 3% higher than the more rigorous 2-D approach from the literature.

Looking at a tube bundle (or a number of parallel plate ducts), the situation is different: No circulation will take place within each one of the single parallel tubes, but in pairs of tubes in that case.

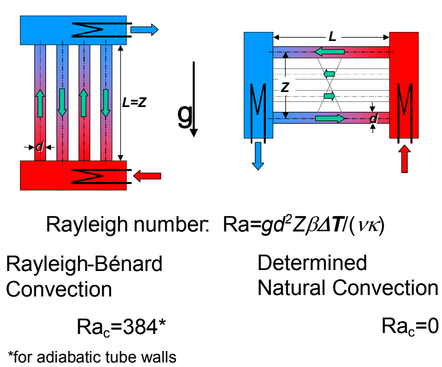

Fig. 4 Natural convection in tube bundles.

Diameter (or gap width) d and length L enter the Rayleigh number as: d^2L, (for L=Z) *i. e.* the Rayleigh numbers of the single channel cases, containing d^4/L have to be multiplied by $(L/d)^2$ to arrive at the correct form of Ra for the bundle.

From this we can find, that $\mathrm{Ra}_{c,\mathrm{single\ tube}}(L/d)^2$ had to be less than $\mathrm{Ra}_{c,\mathrm{tube\ bundle}}$ for circulation within a single tube to occur in a bundle. Using the values from Figs. 2 and 4, namely $\mathrm{Ra}_{c,\mathrm{single\ tube}}$=1087 and $\mathrm{Ra}_{c,\mathrm{tube\ bundle}}$=384, we find $(L/d)^2 < 384/1087$ or $L/d < 0.59$. So, in case of tubes with lengths greater than 0.6 diameters, the single tube internal convection will never happen in a bundle. Using again a 1-D approximate solution, which is certainly sufficient for an engineering approach, the temperature profiles in the tubes of the bundle as a function of the individual flow rates are shown in Fig.5.

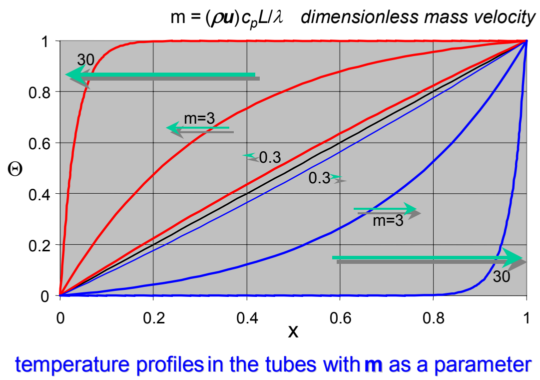

Fig. 5 Temperature profiles in the channels of tube bundles (see Fig. 4) in natural convection as a function of flow rate m

The heat transferred from the hot to the cold side by the combined positive and negative flows in every second channel has been calculated from the same 1-D model by Martin (1992) and the results are shown both for the horizontal, and for the vertical tube bundle as Nusselt numbers versus Rayleigh number in Fig. 6.

For the sake of simplicity, the horizontal bundle in this case consists of only two tubes (or two tube rows) in a vertical distance Z as shown in Fig. 6. For sufficiently high Rayleigh numbers, the heat transfer coefficient is directly proportional to the temperature difference. The heat flux therefore is proportional to the square of ΔT. This of course is the same dependency as found for the single channels (see Fig. 3) in this limit. The difference in behaviour of single tube versus tube bundle can be seen in the fact, that the flow pattern occurring in a bundle differs from that in a single tube, because the greater degree of freedom for the flow, as soon as more than one tube is available for a circulation.

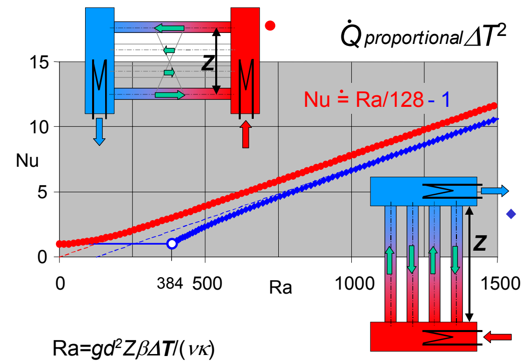

Fig. 6. Nusselt vs. Rayleigh numbers for natural convection in horizontal (upper curve) and vertical (lower curve) tube bundles with adiabatic walls.

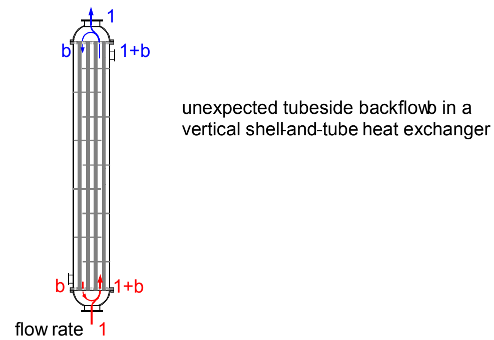

Fig. 7. Unexpected tubeside backflow in a vertical shell-and-tube heat exchanger

MIXED CONVECTION IN BUNDLES

Typically in a classical heat exchanger, we do have forced convection. Natural convection alone is rather unusual, but mixed convection, *i. e.* fluid flow driven by pressure and density gradients, is exactly what happens in every heat exchanger. Usually, however, the effects due to density gradients are neglected in heat exchanger design.

The question, whether natural convection may be safely neglected, is mainly answered on the basis of the well known single channel behaviour. Judging from single channel behaviour, an additional natural convection will usually increase (or only marginally decrease) the heat transfer coefficients compared to forced convection alone (see Aicher and Martin, 1997). That's why neglecting natural convection in general is thought of being justified by staying "on the safe side" in heat exchanger design.

An example from industrial practice taught us, more than ten years ago, that this assumption may be completely wrong (Martin and Pajak, 1994). A single tube pass vertical shell and tube heat exchanger with many segmental baffles on the shell side had been designed and built for heat recovery to operate in counter flow at an efficiency of 91 to 97%.

However, the efficiencies measured in operation were only 61 to 77%. The overall heat transfer coefficient, calculated from the measured temperatures, assuming ideal counter current operation reached only about one third of the design value.

After checking a number of possible reasons for the unexpectedly low performance of that apparatus, we came to the conclusion, that superimposed natural convection might (see Fig. 7) be the reason of such a drastic deviation from the predicted design.

This hypothesis has later been verified

a) by tracer measurements of residence time distributions in the bundle, that actually showed, that backflow existed in some of the tubes.
b) by 1-D model calculations on the flow and temperature distributions in that shell-and-tube apparatus.
c) by changing the flow directions of both tube-side and shell-side fluid in the shell-and-tube apparatus, which indeed resulted in a stable situation avoiding backflow and resulting in the high efficiencies of around 95% as expected from design.

See Martin and Pajak (1994) and Aicher et al. (1999) for details.

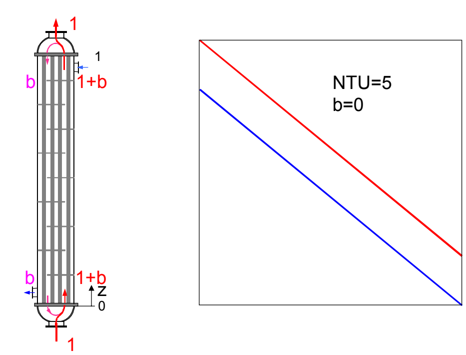

Fig. 8. Temperature distribution without backflow in a vertical shell-and-tube heat exchanger – ideal counter flow.

To demonstrate the effect of unwanted backflow b in a part a of the tubes in a vertical bundle, the model equations from (Aicher et al., 1999) have been used again here in a simpler form. We assume a balanced counter flow heat exchanger with NTU=5. If there is no backflow (as shown in Fig. 8), the temperatures vary linearly with the tube length, and the efficiency would be ε=5/6. The case with backflow can be calculated analytically as the solution of three coupled ordinary differential equations for the temperatures in the tubeside upflow (subscript 1, flow rate=1+b in the fraction 1-a of the tubes), the tube-side backflow (subscript 2, flowrate b in a fraction a of the tubes), and the laterally mixed shell-side flow (subscript 3, assuming no backflow or axial mixing).

The (normalized) temperatures from this analytical solution can be written as:

$$T_j(z)=A_j+B_j\exp(r_2\, z)+ C_j\exp(r_3\, z) \quad j=1,2,3 \qquad (1)$$

Normalized axial coordinate: $0 < z < 1$. The roots of the characteristic equation (r_1=1), r_2, r_3, and the coefficients A_j, B_j, and C_j are listed in the appendix. These coefficients have been obtained from the differential equations and from the boundary conditions:

$$T_1(1)=T_2(1),\ (1+b)T_1(0)=1+b\ T_2(0),\ T_3(1)=0 \qquad (2, 3, 4)$$

The variation of the heat transfer coefficients with individual flowrates has not been taken into account in this calculation for the sake of simplicity. NTU, therefore, has been assumed to be inversely proportional to the flowrate.

Figure 9 shows the results of such a calculation from eqns. (1-4) and the coefficients as given in the appendix. In the paper by Aicher et al. (1999) the calculations were carried out including the effects of flowrates on the heat transfer coefficients (htc), but for a demonstration of the principle, the assumption of constant htc may be acceptable here.

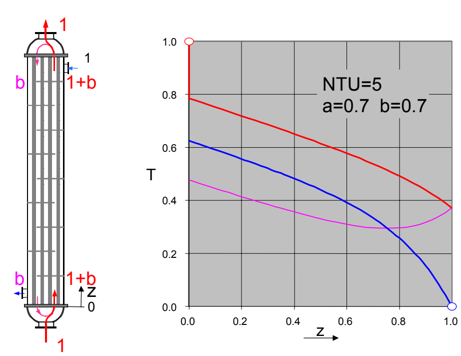

Fig. 9. Temperature distribution with backflow in a vertical shell-and-tube heat exchanger

MAY BACKFLOW IN A TUBE BUNDLE ALSO BE USEFUL?

It has been shown by a number of relatively simple examples, that the results for natural convection in single channels cannot be used directly in bundles. The flow patterns occurring in bundles differ from those in single channels in as far as flow in two opposite directions in these cases is not observed within one channel, but only in separate tubes. From Figs. 8 and 9 we have seen, that backflow in part of the tubes, due to superimposed natural convection, may considerably decrease the performance of a vertical counter current heat exchanger, especially so, if operated in the high NTU, high efficiency range. In order to avoid backflow in these cases, it is to be recommended to have the hot end of the heat exchanger at the top, not the bottom. Internal circulation, however, does increase the local flowrates and, therefore, may increase the overall performance in other cases.

In any case it is clear, that the influence of natural convection should be taken into account in heat exchanger design. Yet, so far nearly nothing on that topic is found in the textbooks as well as in the handbooks of heat transfer and heat exchanger design.

To check, whether backflow in some tubes is possible, we need critical Rayleigh numbers, which depend on the Graetz number in laminar flow, and additionally on the Prandtl number and *d*/*L* in turbulent flow. Only recently, Nickolay (2001) has developed a Ra_I, Ra_{II} vs. Graetz-chart from analytical calculations as well as experimental data with tube bundles.

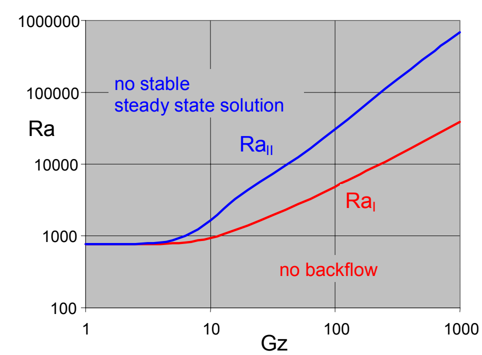

Fig. 10 Nickolay's stability chart for laminar mixed convection in vertical tube bundles.

The Rayleigh number here is defined with d^4/L as the characteristic "volume". For $Ra < Ra_I$ no backflow is observed. For $Ra > Ra_{II}$ there exists no stable solution. In the range between the two limits stable solutions with a certain number of tubes in backflow do exist. The limiting value for low Graetz numbers is Ra_{min}=768. Extensions into the turbulent flow regime have also been calculated by Nickolay (2001).

The curves in Fig.10 have been obtained from the analytic formulae:

$$Ra_I = 32/(\Theta/Gz) \quad (5)$$

$$Ra_{II} = 32/(d\Theta/dGz) \quad (6)$$

and an empirical approximation for the volumetric mean temperature $\Theta = \Theta_{vm}$, obtained from a volume averaged enthalpy content which is different from the caloric mean temperature (corresponding to an average enthalpy flux):

$$\Theta = [1+(24/Gz)^5+(180/Gz)^{5/3}+(3000/Gz)^{5/12}]^{-1/5} \quad (7)$$

The derivative $d\Theta/dGz$ in Ra_{II} can be found analytically from this formula. This approximation has been shown to be in very good agreement to the more rigorous series calculations by Nickolay (2001). Furthermore the experimental observation of backflow in tube bundles verified the existence and importance of these two limiting Rayleigh numbers. When increasing the temperature difference in Ra= $gd^4\beta\Delta T/(L\nu\kappa)$, backflow in one or more tubes is observed, when Ra_{II} is reached. When reducing Ra, backflow remains stable and only vanishes, when Ra_I is reached! In the range between the two critical Rayleigh numbers, the actual flow pattern, and the performance of the apparatus depends on the "history" of its operation before!

A NEW INCENTIVE FOR MORE COMPACT HEAT EXCHANGERS? CONCLUSIONS

Stable operation without backflow can always be reached, if the Rayleigh number stays below Ra_{min}=768.

Scaling down an existing (shell-and-tube) heat exchanger to a smaller volume, when keeping its flowrates (nud^2), efficiency and pumping power constant, requires—in the laminar flow range—that

a) NTU and, therefore, Gz is kept constant $Gz=(nud^2/\kappa)/(\boldsymbol{nL})$, or $\boldsymbol{nL}=c_t$ (8)

b) $\Delta p=32\eta\boldsymbol{L}(nud^2)/(\boldsymbol{nd^4})$, or $\boldsymbol{nd^4}/\boldsymbol{L}=c_h$ (9)

From these scaling laws, one can see, that the length L decreases inversely with the number n of parallel tubes, while, eliminating L from eqns. (8) and (9) shows, that d^2, too, is inversely proportional to n.

So doubling the number of parallel tubes in a bundle, will decrease L, d^2, and the Rayleigh-number Ra by a factor of one half.

Compact heat exchangers are definitely less affected by unwanted backflow problems, than the classical shell-and-tube heat exchangers with tube diameters in the order of centimeters.

NOMENCLATURE

Latin symbols

a	part of tubes in backflow, 1
A	area, m^2
b	backflow ratio, 1
c	constants
d	diameter (gap width), m
Gz	Graetz number, Gz=RePr d/L
L	length, m
m	dimensionless flowrate (Fig. 5), 1
M	shell-side NTU, 1
n	number of parallel tubes, 1
N	tube-side NTU, 1
Nu	Nusselt number, $\alpha d/\lambda$
Pr	Prandtl number, Pr=$\eta c_p/\lambda$
Ra	Rayleigh number, Ra=$gL^3\beta\Delta T/(\nu\kappa)$ (with L^3 replaced by d^4/L or d^2L if appropriate)
Re	Reynolds number, Re=ud/ν
u	flow velocity, m/s
z	coordinate in flow direction, 1

Greek symbols

α	heat transfer coefficient, W/(m^2 K)
λ	thermal conductivity, W/(m K)
κ	thermal diffusivity, m^2/s
η	viscosity, Pas
ν	kinematic viscosity, m^2/s
ρ	density, kg/m^3

Subscripts

1	tube-side upflow
2	tube-side backflow
3	shell-side flow
h	hydrodynamic task
t	thermal task

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APPENDIX

Coefficients, to be used in eqns. (1-4) for the temperatures in a shell-and-tube hx with backflow

$N_1 = N\, a/(1+b)$
$N_2 = N\, (1-a)/b$
$N_3 = M\, a$
$N_4 = M\, (1-a)$
$NA = -(N_1 - N_2 - N_3 - N_4)/2$
$NB = ((N_1 + N_2)^2 + (N_3 + N_4)^2 - 2\,(N_1 + N_2)\,(N_3 - N_4))^{(1/2)}$

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$r_2 = NA + NB/2$
$r_3 = NA - NB/2$
$e_2 = \exp(r_2)$
$e_3 = \exp(r_3)$

$K_{B1} = N_1/(r_2 + N_1)$
$K_{B2} = N_2/(r_2 - N_2)$
$K_{C1} = N_1/(r_3 + N_1)$
$K_{C2} = N_2/(r_3 - N_2)$

$K = e_2\, e_3\, ((K_{B1} + K_{B2}) - (K_{C1} + K_{C2}))$
$\quad - e_2\, (((1+b)\, K_{C1} + b\, K_{C2})\, (K_{B1} + K_{B2}))$
$\quad + e_3\, (((1+b)\, K_{B1} + b\, K_{B2})\, (K_{C1} + K_{C2}))$

$A_1 = A_3$
$A_2 = A_3$
$A_2 = e_2\, e_3\, ((K_{B1} + K_{B2}) - (K_{C1} + K_{C2}))/K$

$B_1 = K_{B1}\, B_3$
$B_2 = -K_{B2}\, B_3$
$B_3 = e_3\, (K_{C1} + K_{C2})/K$

$C_1 = K_{C1}\, C_3$
$C_2 = -K_{C2}\, C_3$
$C_3 = -e_2\, (K_{B1} + K_{B2})/K$