TIME FRACTIONAL TRANSPORT MODEL FOR FLOW THROUGH TIGHT POROUS MEDIA

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Modelling the flow of fluid through tight porous media, such as unconventional hydrocarbon reservoirs, is very challenging and it is a growing sector and must be addressed. Shale gas is found in such tight porous rocks which are characterized by nano-scale size porous networks with ultra-low permeability [1,2].

Here gas non-linear transport models for reservoir simulations of single-phase gas through homogeneous tight rocks, [3], is combined with a fractional calculus method, [4,5], to pose a new time-fractional advection-diffusion transport model [6,7] for the pressure field, \( p = p(x, t) \),

\[
\frac{\partial^\alpha p}{\partial t^\alpha} = L(p) + R(x, t), \quad t > 0, \quad a \leq x \leq b; \quad 0 < \alpha < 1,
\]

and with suitable initial and boundary conditions, where \( L(\cdot) = \frac{\partial}{\partial x} \left( -U(\cdot) + K \frac{\partial}{\partial x}(\cdot) \right) \) is a second order differential operator, \( \frac{\partial^\alpha}{\partial t^\alpha} \) is the Caputo fractional derivative of order \( \alpha \), \( R \) is an source term, and \( b - a \) is the length of the reservoir. In these models, the apparent diffusivity is \( K = K(x; p) \), and the apparent convective velocity is \( U = U(x; p, p_x) \); thus either or both of \( K \) and \( U \) can be non-linear.

We will analyse the solutions for different fractional order \( \alpha \), and compare the solutions with other models and against data where available.

References