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COMBINED EFFECTS OF INTERNAL HEAT GENERATION AND VISCOUS DISSIPATION FOR DOUBLE DIFFUSIVE WITH FORCHHEIMER FLUID MODEL

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ABSTRACT

In this paper, a numerical study using shooting technique is applied for a double diffusive flow for the combined effects of internal heat generation and viscous dissipation over a vertical heated plate under the influence of variable fluid properties is carried out. The governing equations of the physical problem are non-linear and coupled partial differential equations for velocity, temperature and concentration distributions. Using a similarity transformation, the governed PDE's for the physical model transformed into ODE's involving the various non-dimensional parameters of the physical problem. The fluid characteristics of the physical model are discussed under the variable fluid properties for various non-dimensional parameters and the results are compared with earlier published work for a particular cases. It is observed that our results are well agreed with the earlier works in the literature.

KEYWORDS: Double diffusion, internal heat generation, viscous dissipation, porous medium, shooting technique, variable fluid properties.

1 INTRODUCTION

In many decades, the understanding of double diffusive convection plays an important role in many industrial applications in science and technology. In the literature, the study of velocity, temperature and concentration profiles are characterized in terms of many non-dimensional parameter of the physical problem without variation of fluid properties like permeability of the porous media, viscosity of the fluid, the thermal and concentration diffusivity of the fluid. It is been observed that many researchers are studied the convection without incorporating the additional external effects of internal heat generation (IHG) in the temperature equation and Forchheimer term in the Navier-Stokes equation. The effects of IHG and viscous dissipation will helps to forced convection of fluid particles in the fluid region, it turn heat will be transferred from one position to another position [1-3,8-9]. The control of the velocity of the fluid can be appropriately controlled with the quadratic drag called Darcy- Forchheimer effect. This effect dominates over the inertial force and viscous force, which governs by a non-linear relation with the pressure gradient and the velocity of the fluid [7].

In many industrial applications like microwave heating, gas transport, rocket engine, cooling of nuclear reactors etc have variable fluid properties and additional forces. Such a study of these applications do not given much importance in the literature by incorporating various physical changes in the fluid properties and additional effects. Hence, the main of this work is to understand the above application problems by developing a mathematical model. Here, we study the combine effects of internal heat generation, viscous dissipation and Forchheimer model for double diffusive mixed convection heat and mass transfer with exponential changes in fluid variables of permeability, porosity, thermal conductivity and solutal diffusivity of the physical system.

NOMENCLATURE

C_P	Specific heat at constant pressure
C_b	Empirical constant of the second-order resistance term due to inertial effect
C_w	Concentration at the plate
C_∞	Concentration far away from the plate
E	Eckert number
f	Non-dimensional stream function

g Acceleration due to gravity

G_r Grashof number

$k(y)$ Permeability of the porous medium

k_0 Permeability of the porous medium at the edge of the boundary layer

k Thermal conductivity

$\Pr$ Prandtl number

Re Reynolds number

T Temperature of the fluid near the plate

T_w Temperature at the plate

T_∞ Ambient temperature

u, v Velocity components along x and y directions

U_0 Free stream velocity

x, y Co-ordinates axes along and perpendicular to the plate

$\alpha(y)$ Thermal diffusivity

α^* Ratio of viscosities

α_0 Thermal diffusivity at the edge of the boundary layer

β Coefficient of volume expansion

β^* Local inertial parameter

$\varepsilon(y)$ Porosity of the saturated porous medium

ε_0 Porosity of the saturated porous medium at the edge of the boundary layer

η Dimensionless similarity variable

θ Dimensionless temperature

μ Viscosity of porous medium

$\overline{\mu}$ Effective Viscosity of the fluid

ν Kinematics viscosity of the fluid

ψ Stream function

ρ Density of fluid

σ^* Ratio of thermal conductivity of the solid to the liquid

σ Permeability parameter

τ' Skin friction

Nu Nusselt number

Sh Sherwood number

Ri Richardson number

2 MATHEMATICAL FORMULATION

Here, the physical model is a two-dimensional, steady, laminar, incompressible double diffusive convective fluid flow along a semi-infinite vertical plate in a saturated porous medium with the variation of fluid properties like permeability of the porous media, viscosity of the fluid, the thermal and concentration diffusivity of the fluid. For the importance of industrial applications, the additional effects like Forchheimer model, IHG and viscous dissipation terms are included in the momentum and energy equations. The x- coordinate is taken along a vertical semi-infinite plate and y -coordinate is perpendicular to it. Let U_0 be the free stream velocity of the fluid in the direction and the gravitational force g is considered opposite the free stream velocity. A constant temperature T_W and constant concentration C_W are maintained at the semi-infinite vertical plate and or greater than the temperature T_∞ and the concentration C_∞ far away from the vertical semi-infinite plate (i.e. $T_W > T_\infty$ and $C_W > C_\infty$). By considering boundary layer theory approximations and its assumptions, the governing equations of the physical system for the fluid flow can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \beta_T (T - T_\infty) - g\beta_C (C - C_\infty) + \frac{\overline{\mu}}{\rho}\frac{\partial^2 u}{\partial y^2} - \frac{\mu}{\rho}\frac{\varepsilon(y)}{k(y)}u - C_b\frac{\varepsilon^2(y)}{\sqrt{k(y)}}u^2, \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left(\alpha(y)\frac{\partial T}{\partial y}\right) + q''' + \frac{\overline{\mu}}{\rho . C_P}\left(\frac{\partial u}{\partial y}\right)^2, \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = \frac{\partial}{\partial y}\left(\gamma(y)\frac{\partial C}{\partial y}\right). \tag{4}$$

The associated boundary conditions for the physical model are

$$\begin{aligned} &u = 0 \;: v = 0 \;\; T = T_w \;\; C = C_w \;\; at \;\; y = 0, \\ &u = U_0 : v = 0 \;\; T = T_\infty \; C = C_\infty \;\; as \;\; y \to \infty. \end{aligned} \tag{5}$$

Here the physical quantities which are appear in the equations are of their standard meanings. where q''' is the exponential form of Internal Heat generation, $k(y), \varepsilon(y), \alpha(y)$ and $\gamma(y)$ are the variable permeability, porosity, thermal and solutal diffusivity of the porous medium. In order to solve the above nonlinear partial differential equations, we introduce the stream function with a dimensionless similarity variable η. where $\eta = \frac{y}{x}\left(\frac{U_0 x}{\nu}\right)^{1/2}$,

$$u = U_0 \, f'(\eta); \;\; v = -\frac{1}{2}\sqrt{\frac{\nu U_0}{x}}\left(f(\eta) - \eta . f'(\eta)\right) \tag{6}$$

Also, the variable permeability $k(y)$, the porosity $\varepsilon(y)$, the effective thermal diffusivity $\alpha(y)$ and the solutal diffusivity $\gamma(y)$ are defined as

$$k(\eta)=k_0(1+d\,e^{-\eta}),\quad \varepsilon(\eta)=\varepsilon_0(1+d^*e^{-\eta}),$$
$$\alpha(\eta)=\alpha_0\left(\varepsilon_0(1+d^*e^{-\eta})+\sigma^*\{1-\varepsilon_0(1+d^*e^{-\eta})\}\right),$$
$$\gamma(\eta)=\gamma_0\left(\varepsilon_0(1+d^*e^{-\eta})+\gamma^*\{1-\varepsilon_0(1+d^*e^{-\eta})\}\right). \quad (7)$$

where k_0, ε_0, α_0 and γ_0 are the initial amplitudes of permeability, porosity, thermal conductivity and solutal diffusivity of the porous medium, σ^* is the ratio of thermal conductivity of solid to the conductivity of the fluid, γ^* is the ratio of the thermal diffusivity of solid to the diffusivity of the fluid. Also d and d^* are treated as initial amplitudes of the Uniform permeability (UP) and Variable permeability (VP) cases. Here, we choose these constants as 3.0 and 1.5 respectively for VP case, in case of UP we choose the constants as zero. Rewriting equations (2),(3) and (4) using (6) and (7), the ordinary differential equations becomes

$$f'''=-\frac{1}{2}f\,f''-\frac{Gr}{\mathrm{Re}^2}(\theta+N\phi)-\frac{\alpha^*}{\sigma\,\mathrm{Re}}$$
$$\frac{(1+d^*e^{-\eta})}{(1+d\,e^{-\eta})}(1-f')-\frac{\beta^*(1+d^*e^{-\eta})^2}{(1+d\,e^{-\eta})^{1/2}}\left(1-f'^2\right), \quad (8)$$

$$\theta''=\frac{-\left(\frac{1}{2}\Pr f\,\theta'+\Pr.E\,f''^2+\frac{1}{2}\Pr e^{-\eta}+\varepsilon_0 d^*e^{-\eta}(\sigma^*-1)\theta'\right)}{\varepsilon_0+\sigma^*(1-\varepsilon_0)+\varepsilon_0\,d^*e^{-\eta}(1-\sigma^*)}, \quad (9)$$

$$\phi''=\frac{-\left(\frac{1}{2}Sc\,f\,\phi'+\varepsilon_0\,d^*e^{-\eta}(\nu^*-1)\,f_7\right)}{\varepsilon_0+\nu^*(1-\varepsilon_0)+\varepsilon_0\,d^*e^{-\eta}(1-\nu^*)}, \quad (10)$$

and the boundary conditions in terms of $f,\theta\,and\,\phi$ are

$$f=0\quad f'=0\quad \theta=1\quad \phi=1\quad at\quad \eta=0,$$
$$f'=1\quad \theta=0\quad \phi=0\quad as\quad \eta\to\infty. \quad (11)$$

where the non-dimensional parameters of the physical problem are $\beta^*=\frac{F\varepsilon_0^{\,2}x}{k_0^{1/2}}$ is the local inertial parameter, $\Pr=\frac{\bar{\mu}}{\rho\alpha_0}$ is the Prandtl number, $Sc=\frac{\bar{\mu}}{\rho\gamma_0}$ is the Schmidt number, $\alpha^*=\frac{\mu}{\bar{\mu}}$ is the ratio of viscosities, $E=\frac{U_0^{\,2}}{C_P(T_W-T_\infty)}$ is the Eckert number, $\sigma=\frac{k_0}{x^2\varepsilon_0}$ is the local permeability parameter, $N=\frac{\beta_C(C_w-C_\infty)}{\beta_T(T_w-T_\infty)}$ is the Buoyancy ratio, $\mathrm{Re}=\frac{U_0\,x}{\nu}$ is the local Reynolds number, $Gr_T=\frac{g\,\beta_T(T_w-T_\infty)x^3}{\nu^2}$ is the local Thermal Grashof number, $Gr_C=\frac{g\,\beta_C(C_w-C_\infty)x^3}{\nu^2}$ is the local solutal Grashof number and $Ri=\frac{Gr}{\mathrm{Re}^2}$ is the Richardson number which is the convection parameter. The Skin friction, the rate of heat and mass transfers for the vertical plate are given by

$$\tau=-f''(0)/\sqrt{\mathrm{Re}},$$
$$q_W=-k\left(\frac{\partial T}{\partial y}\right)_{y=0},\quad q_m=-D\left(\frac{\partial C}{\partial y}\right)_{y=0}. \quad (12)$$

The non dimensional local Nusselt and Sherwood numbers are given by

$$\frac{Nu_x}{\mathrm{Re}_x^{1/2}}=-\theta'(0),\qquad \frac{Sh_x}{\mathrm{Re}_x^{1/2}}=-\phi'(0). \quad (13)$$

3 NUMERICAL METHOD

The nonlinear system of coupled ODE's (8)-(10) with appropriate boundary conditions (11) are solved numerically of the physical model, using a suitable numerical shooting technique by Runge-Kutta-Fehlberg algorithm and Newton-Raphson method. Here, the selection of an appropriate finite value of '∞' to be determined is the most important aspect by using Newton-Raphson method algorithm. In this technique, we begin our computations by choosing a suitable initial guess value for the initial conditions in replacement for the given boundary condition of the physical model for velocity, temperature and concentration conditions.(i.e., $f''(0),-\theta'(0)$ and $-\phi'(0)$). To select '∞', we begin with some initial guess value and solve the problem with some particular set of parameters to $f''(0),-\theta'(0)$ and $-\phi'(0)$. If the guess values are correct by satisfying the boundary conditions we terminate the computation otherwise we repeat the iterative process for the better guess values using Newton-Raphson method for the accuracy of 10^{-6} by taking the step size h=0.001 for the convergence criteria.

4 RESULTS AND DISCUSSION

In this numerical computation of the physical problem, the skin-friction coefficient, the Nusselt and Sherwood numbers, which are proportional to $f''(0)$, $-\theta'(0)$ and $-\phi'(0)$ are obtained and their behaviors are presented graphically for the importance of industrial applications. As a particular case, our results are compared on velocity, temperature and concentration distributions with the earlier published works carried out by Mohammadein and El-Shaer[5], and Nalinakshi et al[10] by choosing the non-dimensional parameters N=0, $\beta^* = 0$ and in the absence of concentration distribution for the VP and UP cases. The results are found good agreement for the particular values of the non-dimensional parameters of the physical system.

Numerical computations has been carried out for the mixed double diffusive convection for velocity, temperature and concentration for the study of combined effects of IHG, viscous dissipation and Forchheimer model under the influence of variable fluid properties.

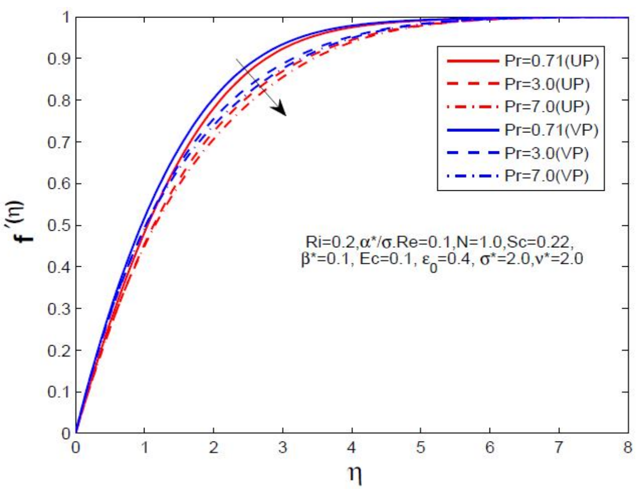

Fig.1: Velocity characteristics for different values of Pr .

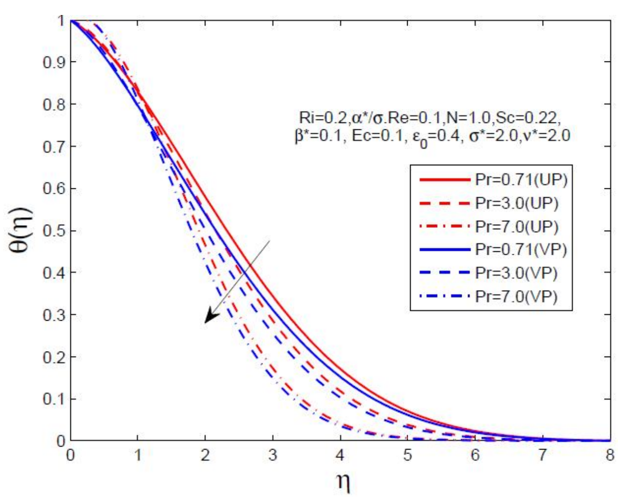

Fig. 2.: Temperature characteristics for different values of Pr

Fig.1, depicts the characteristics of velocity for different values of Pr in the case of UP and VP. From the figure seen that with an enhancement in the magnitude of Pr, there is a significant reduction for the flow. This is due to the higher viscosity present in the fluid flow. This is a well-known agree with the physical property of the fluid. Similar significant behaviour is seen for the temperature profile in Fig.2. There is a slight variation on concentration for the enhancement of Pr both in the cases of UP and VP. Since concentration is not a direct function Pr. For want of space the graphical representation is not mentioned.

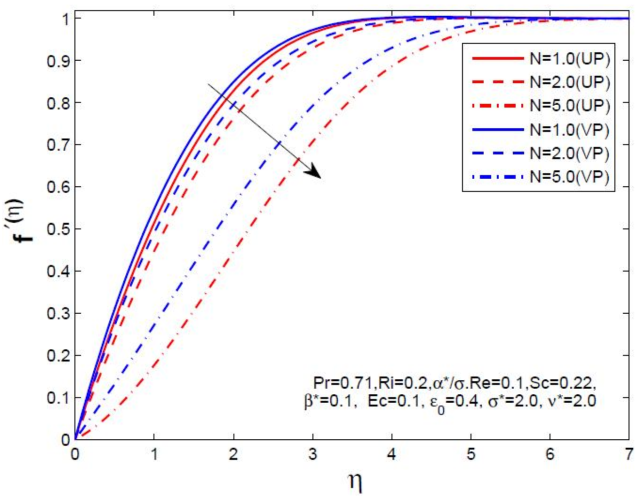

Fig.3: Velocity characteristics for different values of N.

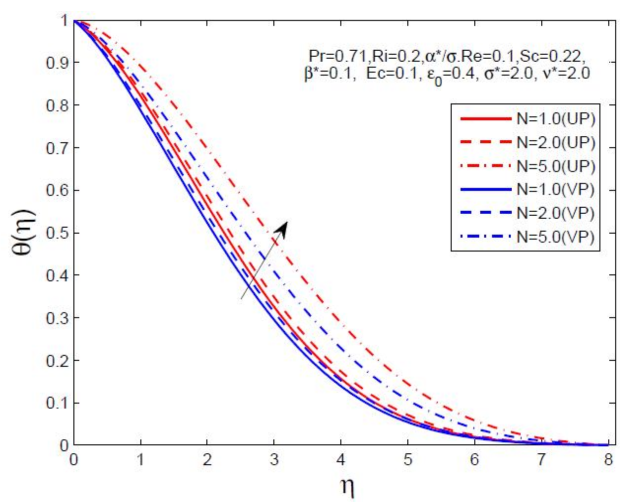

Fig. 4: Temperature characteristics for different values of N.

Due to the variations of the Buoyancy ratio N, there is an opposite behavior can be observed in the case of velocity and temperature which are shown in figs.3 and 4 respectively. This effect is due to the temperature and concentration gradient between the variable plate and far away from the plate. There is a significant reduction on velocity over the buoyancy ratio. It is also observed that, the effect of Buoyancy ratio is more on velocity in the case VP compared to that of UP.

The concentration distributions for various values of Schmidt number Sc have been depicted in Fig.5 and observed that distributions will reduce with the fluid motion. The amplication of Sc is due to the reduction in the solutal diffusivity of the fluid.

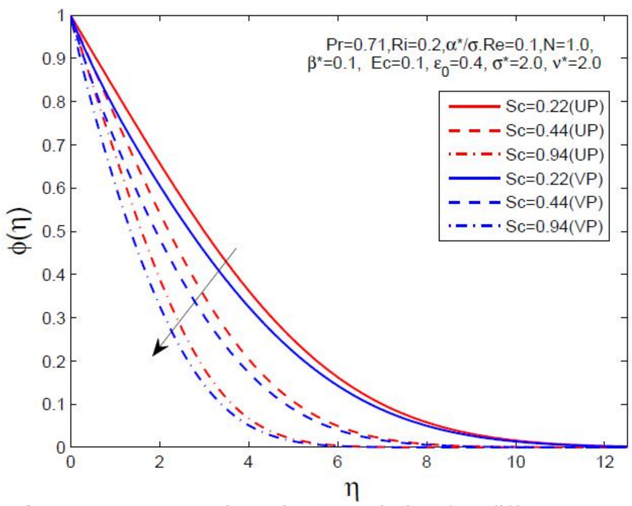

Fig. 5: Concentration characteristics for different values of Sc.

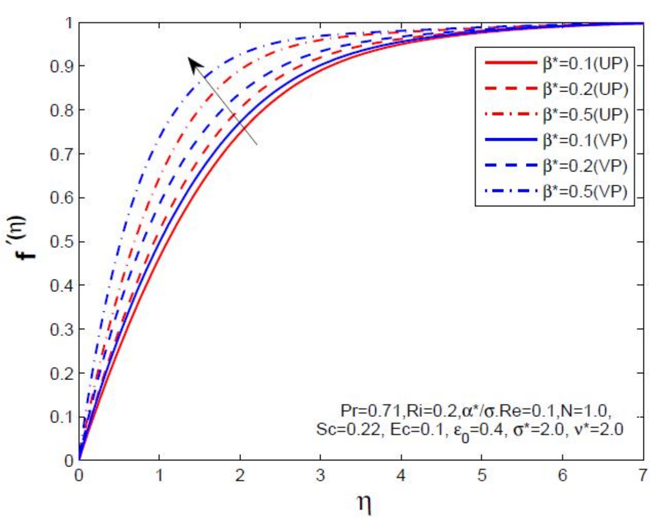

Fig.6: Velocity characteristics for different values of β^*.

For the importance of many practical applications as mentioned above the numerical computation has been carried out to see the effect of local inertial parameter β^* on velocity, temperature and concentration. For want of space β^* on velocity is described graphically from Fig.6. It is observed that the effect of β^* enhances the velocity profiles due to momentum of the inertial acceleration of the fluid motion but an opposite behavior can be observed in the case of temperature and concentration due to the enhancement of the motion of the fluid.

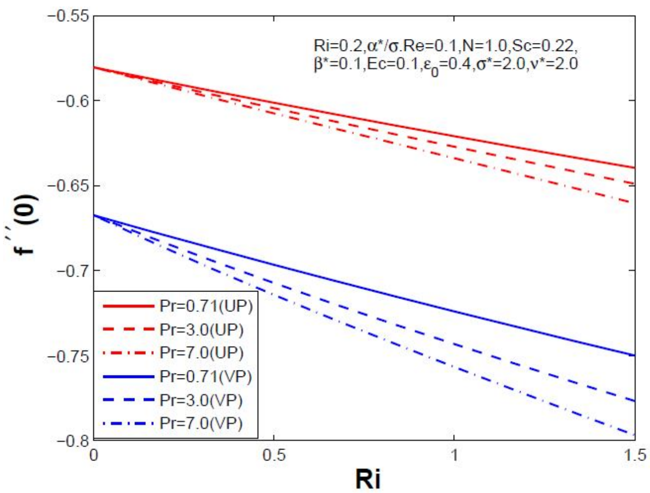

Fig. 7: Effects of Skin friction vs Ri for various values of Pr.

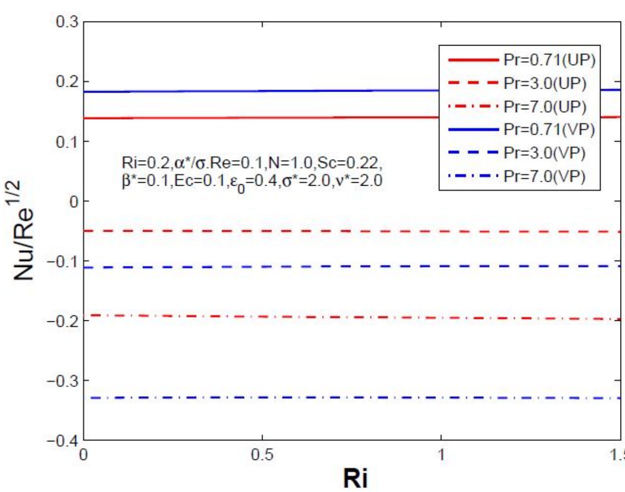

Fig. 8: Effects of local Nusselt number vs Ri for various values of Pr.

For the industrial applications, the Skin friction, Nusselt and Sherwood number's are computed as a function of Richardson number Ri for various values of Prandtl number which are depicted in figures 7, 8 and 9 respectively. There is a significant reduction on skin friction for the enhancement of Prandtl number Pr due to increase of viscosity near the vertical plate (or) the boundary layer thickness. Hence an opposite behavior can be seen in the case of Nusselt number (or) rate of heat transfer and

Sherwood number. Also from all the above figures, it is observed that the variation in significantly more in the case of VP compared to that of UP case.

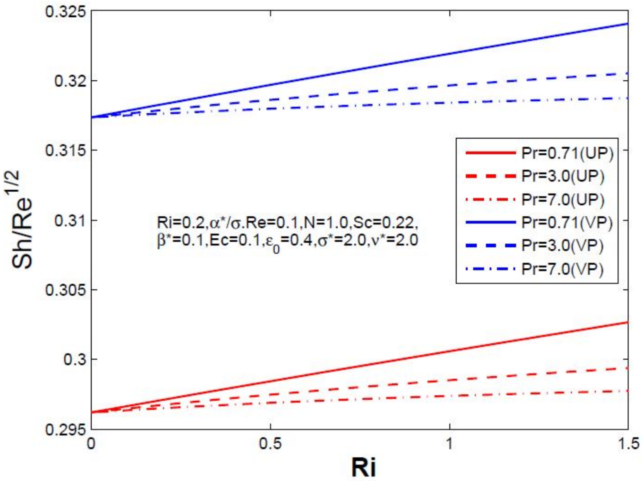

Fig. 9: Effects of Sherwood number vs Ri for various values of Pr.

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