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Internal Heat Generation (IHG) Effect on Mixed Convection Heat and Mass Transfer over a vertical heated plate with Soret and Dufour effects

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ABSTRACT

A numerical study and analysis of internal heat generation effect of double diffusive mixed convection flow of a viscous incompressible fluid over a heated plate by varying fluid properties with Soret and Dufour effects is been carried out. Lapwood-Forcheimer Brinkman extended Darcy model is been governed for the boundary layer flow in the porous medium. In this analysis, the technique used is the shooting technique which involves Runge-Kutta-Fehlberg scheme followed by Newton-Raphson method where the governing equations are non-linear and highly coupled.Using the similarity transformations the equations are solved to obtain distributions in terms of non-dimensional parameters involved in the physical configuration. The features of fluid flow, heat and mass transfer characteristics are analyzed by in detail to interpret the effects of various significant parameters of the problem. Local surface temperature becomes more significant with effect of the Dufour parameter and concentration increases due to Soret effect. The viscous drag, rate of heat and mass transfer effected by the pertinent parameters are depicted graphically. The obtained results are in good agreement when compared with previously published work of the problem.

Key Words: Heat and Mass transfer, boundary layer, IHG, Soret and Dufourparameters.

I. INTRODUCTION

During the past decades a great effort has been devoted to understand the convective instability in a fluid-saturated porous layer subject to various additional effects depending on the applications in a variety of geometrical and technological problems. The study of internal heat generation with Soret and Dufour effects is important in several physical problems such as fluids undergoing exothermic and endothermic chemical reaction.Due to strong IHG the plate surface temperatures exceed the temperature of the fluid on the lower surface of the plate and the direction of heat flow is reversed leading to enhancing the melting and impeding the freezing. Literatures [1]-[3] are based on the studies relating to internal heat generation.The study of double diffusive mixed convection with internal heat generation is studied in detail pertaining to different physical systems. Very less literature is available for the study of mixed convection heat and mass transfer with internal heat generation by varying fluid properties.

Soret and Dufour effects are more significant Whenlower density species are introduced at a surface and density differences arise in the fluid flow regime.The thermal-diffusion (Soret) and diffusion-thermo (Dufour) effects are interesting macroscopically physical phenomenon in fluid mechanics. Studies made in the literature {[4]-[7]} investigated numerically the effect of double-diffusive natural convection of water in a partially heated enclosure with Soret and Dufour coefficients around the density maximum.

All the above cited literature of thermal-diffusion and diffusion-thermo effects are either confined to fluid flow or porous medium, vertical heated plate or other geometries, cross diffusion or double diffusion, Darcy or non-Darcian porous medium, with or without viscous dissipation effect and with or without variable fluid properties, which has wide applications in engineering processes. Now an attempt is made to study the Heat and Mass transfer in a vertical heated plate with IHG and Soret and Dufour effects by varying the fluid properties such as variable porosity, permeability, thermal conductivity and solutal diffusivity.

The main objective of this work is to study and analyse the effect of IHG on double diffusive mixed convection over a heated plate with Soret and Dufour effects embedded in a sparsely packed porous medium incorporating the variable properties such as porosity, permeability, thermal conductivity and solutal diffusivity. Using similarity transformations the governing equations which are highly coupled non-linear are transformed and applied the numerical method called Shooting technique to obtain the boundary layer profiles for various physical parameters. The local Nusselt number and local Sherwood number variations are also shown graphically. The computed results obtained are verified for the accuracy of the method used under the limiting conditions which agree well with the existing ones.

2. MATHEMATICAL FORMULATION

Steady state, laminar, 2-dimensional,viscous, incompressible fluid flow over a semi-infinite vertical heated plate maintained at a uniform temperature and concentration T_W and C_W greater than the free stream values existing far from the plate is considered. The y-coordinate is normal to the plate and x-coordinate is measured from the leading edge of the plate. The governing basic equations by considering the theory of boundary layer effect and with the assumptions can be written in the form:

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho_o}\frac{\partial p}{\partial x}+g\beta_T(T-T_\infty) -g\beta_c(C-C_\infty)+\frac{\bar{\mu}}{\rho_o}\frac{\partial^2 u}{\partial y^2}-\frac{\mu}{\rho_o}\frac{\varepsilon(y)}{k(y)}u-C_b\frac{\varepsilon^2(y)}{\sqrt{k(y)}}u^2 \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left(\alpha(y)\frac{\partial T}{\partial y}\right)+q''' +\frac{\bar{\mu}}{\rho_o C_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{D_m K_T}{C_s C_p}\frac{\partial^2 C}{\partial y^2} \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=\frac{\partial}{\partial y}\left(\gamma(y)\frac{\partial C}{\partial y}\right)+\frac{D_m K_T}{T_m}\frac{\partial^2 T}{\partial y^2}, \tag{4}$$

All the physical quantities have their standard meaning. The following are the boundary conditions for such a physical configuration.

$$u=0,\quad v=0,\quad T=T_W,\quad C=C_W \quad \text{at} \quad y=0,$$

$$u=U_o,\quad v=0,\quad T=T_\infty,\quad C=C_\infty \quad \text{as} \quad y\to\infty. \tag{5}$$

Equations (1) - (4) are highly coupled nonlinear partial differential equations, the dimensionless variables f , $\theta,\phi,$ and q''' and as well as the similarity variable η with stream function ψ are introduced to solve them. (see Nalinakshi et al [9]):

$$\eta=\left(\frac{y}{x}\right)\left(\frac{U_o x}{\nu}\right)^{1/2},\quad \psi=\sqrt{\nu U_o x}\,f(\eta),\quad \theta=\frac{T-T_\infty}{T_W-T_\infty},$$

$$\phi=\frac{\mathrm{C}-\mathrm{C}_\infty}{\mathrm{C_W}-C_\infty},\quad q'''=\frac{U_o(T_W-T_\infty)}{2x}e^{-\eta} \tag{6}$$

The variable permeability $k(\eta)$, the variable porosity $\varepsilon(\eta)$, variable effective thermal conductivity $\alpha(\eta)$ and the variable effective solutal diffusivity $\gamma(\eta)$ are defined as,

$$k(\eta)=k_o(1+de^{-\eta})\ ;\ \varepsilon(\eta)=\varepsilon_o(1+d^*e^{-\eta});$$
$$\gamma(\eta)=\gamma_o\left[\varepsilon_o(1+d^*e^{-\eta})+\gamma^*\left\{1-\varepsilon_o(1+d^*e^{-\eta})\right\}\right];$$
$$\alpha(\eta)=\alpha_o\left[\varepsilon_o(1+d^*e^{-\eta})+\sigma^*\left\{1-\varepsilon_o(1+d^*e^{-\eta})\right\}\right] \tag{7}$$

$k_o,\varepsilon_o,\alpha_o$ and γ_o are the permeability, porosity, thermal conductivity and solutal diffusivity at the edge of the boundary layer respectively.

Equations (2) - (4) using Eqs. (6) and (7) are transformed to the local similarity equations as

$$f'''+\frac{1}{2}f\,f''+\frac{Gr}{\mathrm{Re}^2}(\theta-N\phi)+\frac{\alpha^*(1+d^*e^{-\eta})}{\sigma\,\mathrm{Re}(1+de^{-\eta})}(1-f') +\beta^*\frac{(1+d^*e^{-\eta})^2}{(1+de^{-\eta})^{1/2}}\left(1-f'^2\right)=0 \tag{8}$$

$$\theta''=\frac{-\left(\frac{1}{2}\right)\Pr\theta' f-\Pr E f''^2-\frac{1}{2}\Pr e^{-\eta}-\varepsilon_o d^* e^{-\eta}\left(\sigma^*-1\right)\theta'-\Pr D_f\phi''}{\varepsilon_o+\sigma^*(1-\varepsilon_o)+\varepsilon_o d^* e^{-\eta}\left(1-\sigma^*\right)}, \tag{9}$$

$$\phi''=\frac{-\left(\frac{1}{2}\right)Scf\phi'-\varepsilon_o d^* e^{-\eta}(\gamma^*-1)\phi'+ScS_r\theta''}{\varepsilon_o+\gamma^*(1-\varepsilon_o)+\varepsilon_o d^* e^{-\eta}(1-\gamma^*)} \tag{10}$$

$D_f=\dfrac{D_m k_T(C_W-C_\infty)}{c_S c_p(T_W-T_\infty)}$ is the Dufour number,

$Sr=\dfrac{D_m K_T(T_W-T_\infty)}{\nu\,\mathrm{T_m}(C_W-C_\infty)}$ is the Soret number.

The transformed boundary conditions are:

$$f=0,\quad f'=0,\quad \theta=1,\quad \phi=1 \quad \text{at} \quad \eta=0$$

$$f'=1,\quad \theta=0,\quad \phi=0 \quad \text{as} \quad \eta\to\infty \tag{11}$$

An expression for the skin friction τ , Nusselt number Nu and Sherwood number Share in the form $\tau=-f''(0)/\sqrt{\mathrm{Re}}$, $Nu=-\sqrt{\mathrm{Re}}\,\theta'(0)$ and $Sh=-\sqrt{\mathrm{Re}}\,\phi'(0)$. which are very important for many practical applications.

3. METHOD OF SOLUTION

The governing equations arising pertaining to the physical system due to heated plate are highly coupled nonlinear partial differential equations which are difficult to solve. In order to solve them, we reduce the partial differential equations into ordinary differential equations and convert into seven simultaneous equations of first order for seven unknowns and solve using the numerical technique called shooting technique which involves Runge-Kutta-Fehlberg scheme and Newton-Raphson method for the accuracy of 10^{-6}.

4. RESULTS AND DISCUSSION

The paper highlights the Numerical results for the effects of Internal heat generation and variable fluid properties like permeability, porosity, thermal conductivity and solutal diffusivity for Mixed convection heat and mass transfer over a vertical heated plate with Soret and Dufour effects.

From the table1, it is observed that for various values of non-dimensional parameters of the physical problem considered, the velocity, temperatures and concentration values are tabulated for far away from the plate. We observe that the variable permeability dominates more when compared to uniform permeability. The increase in Dufour number (or decrease in Soret number) leads to velocity and temperature of the fluid increases whereas the concentration of the fluid increases with decrease in Dufour number (or increase in Soret number).

Our results will coincide with earlierwork of Mohommadein El-shaer [8] for the particular case in the absence of buoyancy ratio (N = 0), mass transfer, IHG and soret and dufour effects. Similarly our results also coincide with Nalinakshi et al [9] for the absence of variable solutal diffusivity and soret and dufour effects. For want of space the results are not mentioned here.

Pertaining to different arising physical parameters the results are depicted graphically in Figs. (1) - (8). The contribution of the concentration gradients to the thermal energy flux in the flow and temperature gradients inducing significant mass diffusion effects signified by Dufour and Soret number respectively is observed in the velocity throughout the boundary layer as shown in Fig.1. For D_f > 1, a distinct velocity overshoot exists near the plate, and then after the boundary layer slowly increases towards the wall.

The effect of various values of N which defines the ratio of concentration buoyancy force to thermal buoyancy force parameters can be observed from Figs. 2and 3. The buoyancy ratio parameter N decreases the boundary layer. The effects of N on the temperature and concentration profiles are very small due to the fact that the physical parameter N appears only in the momentum equation. Increase in the value of N decreases the temperature and concentration profiles.

The effect of different values of second order resistance β^* which is expressed with the Forchheimer term and the ratio of porosity to the permeability can be observed from Fig. 4.Increase in β^* leads to increase in the velocity profiles. At β^* = 0.0, the boundary layer shows the linear behavior and as β^* increases the boundary layer shows the exponential form reaching far away from the plate.

The mixed convection parameter i.e., the Richardson number Gr/Re^2 effects are shown in Fig. 5. Increase in the buoyancy force leads to higher Gr/Re^2 which accelerates the fluid leads the velocity closer to the vertical heated plate, and the free convection currents from the heated plate are carried away to the free stream with a downward acceleration acting leading to the understanding that varying the fluid properties is more dominant than the uniformity.

The effects of Prandtl number Pr on velocity profiles are displayed in Fig. 6and show the significant overshoot in the velocity profiles near the wall for lower Prandtl number than compared with higher Pr. The magnitude of the overshoot decreases as the Prandtl number increases. The effect is more in low Prandtl number $(\mathrm{Pr} = 0.71)$ due to the low viscosity of the fluid, which increases the velocity within the boundary layer.

Figs. 7 and 8 depicts, the variations of the rate of heat transfer Nu and Sherwood numberSh as a function of D_f for various values of S_r are shown in Figure 7, describes the behavior of Nusselt number with changes in the values of D_f and S_r. Increase in D_f leads to increase in Nusselt number for every increasing values of S_r for both uniform and variable permeability cases. From Fig. 8, it is observed that the values of local Sherwood number increase with increase in the value of S_r for both UP and VP cases. Also, higher the value of S_r, greater its effect on Sherwood number for all values of D_f as observed in the Fig.8 i.e., the effect of Dufour number on Sherwood number is appreciable in the solutal boundary layer, which increases with increase in the Dufour number.

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Table 1 Results for $f''(0)$, $-\theta'(0)$ and $-\phi'(0)$ for $\Pr = 0.71$, $Sc = 0.22$, $Ec = 0.1$, $\varepsilon_o = 0.4$ for Uniform Permeability (UP) and Variable Permeability (VP) cases.

D_f, S_r	N	σ^*	Gr/Re^2	$\alpha^*/\sigma\mathrm{Re}$	β^*	Uniform Permeability (UP)			Variable Permeability (VP)		
						$f''(0)$	$-\theta'(0)$	$-\phi'(0)$	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
2.00,0.03	1	2	0.2	0.1	0.0	0.4493	0.37845	0.37750	0.4523	0.3814	0.37850
					0.1	0.4645	0.39456	0.39000	0.4745	0.4045	0.39356
					0.5	0.5545	0.43878	0.42950	0.5645	0.4417	0.43650
					1.5	0.6478	0.50703	0.49990	0.6578	0.5100	0.50990
				0.0	0.1	0.4234	0.35134	0.34802	0.4334	0.3613	0.35002
				0.1		0.4645	0.39456	0.39000	0.4745	0.4045	0.39356
				0.5		0.5345	0.45825	0.45721	0.5545	0.4634	0.46111
			0.0	0.1	0.1	0.3978	0.27980	0.27370	0.3678	0.2913	0.28178
			0.2			0.4645	0.39456	0.39000	0.4745	0.4045	0.39356
			1.0			0.7156	0.51890	0.50005	0.7200	0.5029	0.49105
			2.0			0.8435	0.54125	0.53850	0.8512	0.5402	0.53120
		2.0	0.2	0.1	0.1	0.4645	0.39456	0.39000	0.4745	0.4045	0.39356
		4.0				0.4956	0.41178	0.40801	0.5015	0.4232	0.41001
		6.0				0.5524	0.43178	0.42589	0.5698	0.4432	0.43316
	0.0	2.0	0.2	0.1	0.1	0.3994	0.31245	0.31001	0.4012	0.3239	0.32001
	1.0					0.4645	0.39456	0.39000	0.4745	0.4045	0.39356
	5.0					0.7456	0.44189	0.44001	0.7312	0.4500	0.44841
	10.0					0.8890	0.49105	0.49155	0.8950	0.5090	0.49855
2.00,0	1.0	2.0	0.2	0.1	0.1	0.4645	0.39456	0.39000	0.4745	0.4045	0.39356
1.00,0						0.4811	0.40025	0.39950	0.4831	0.4134	0.40013
0.40,0						0.4945	0.41670	0.41202	0.4985	0.4200	0.41894
0.15,0						0.5289	0.42189	0.42004	0.5316	0.4299	0.42789
0.06,1						0.5489	0.43456	0.43001	0.5578	0.4387	0.43398
0.03,2						0.5656	0.45178	0.44901	0.5698	0.4637	0.45765

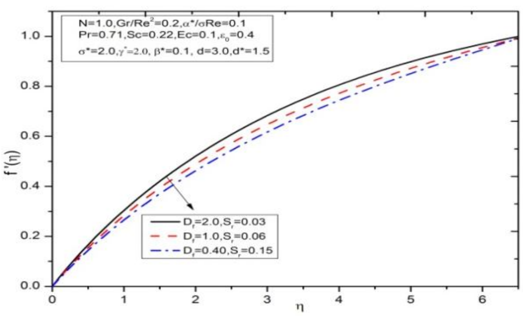

Fig. 1 Velocity profiles for different values of S_r and D_f numbers for VP case

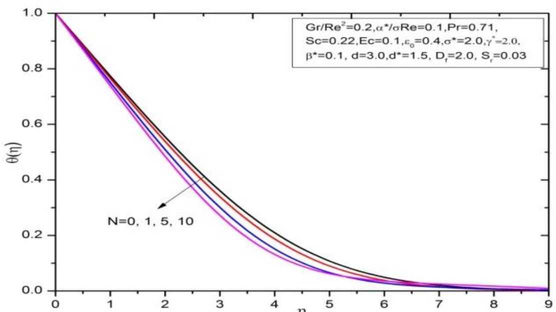

Fig.2 Temperature profiles for different values of buoyancy ratio N for VP case

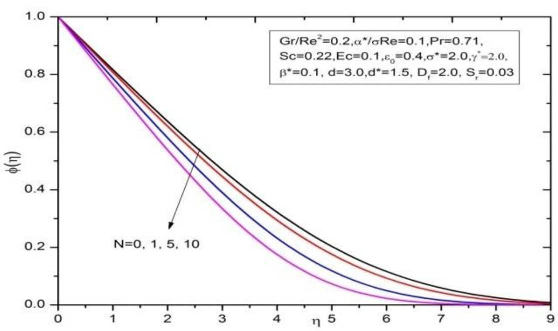

Fig. 3 Concentration profiles for different values of buoyancy ratio N for VP case

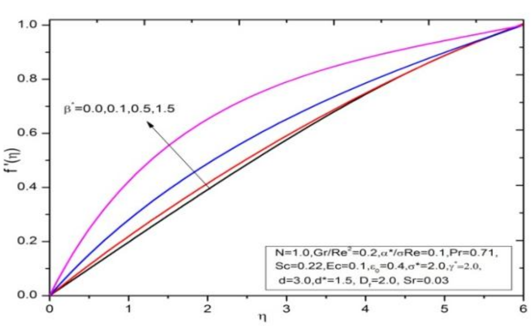

Fig. 4 Velocity variations for various values of second order resistance $\beta*$ for VP case

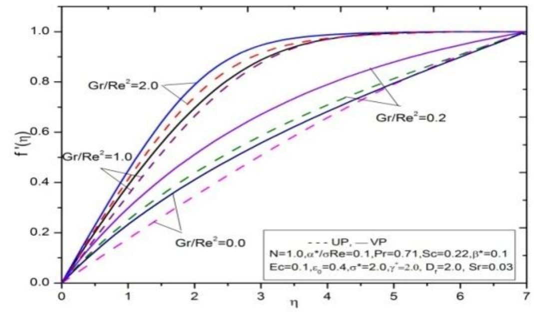

Fig. 5 Velocity distributions for different values of Gr/Re^2 for UP and VP cases

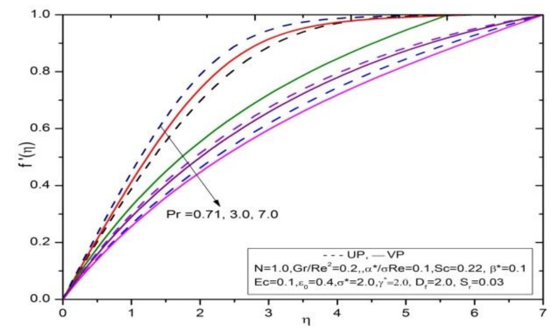

Fig. 6 Velocity distributions for different Pr values for UP and VP cases.

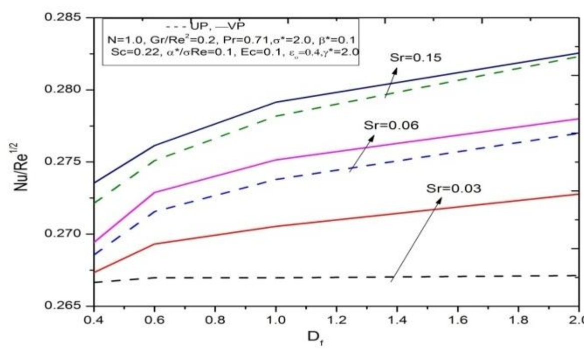

Fig. 7 Variations for UP and VP cases of Nusselt number with D_f for various values of Sr.

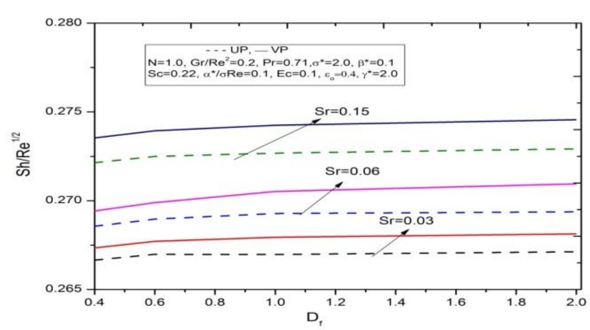

Fig. 8 Variations for UP and VP cases of Sherwood number with D_f for various values of Sr

REFERENCES

[1] MAKINDE O.D., & AZIZ A., “Mixed convection from a convectively heated vertical plate to a fluid with internal heat generation”, J. Heat Transfer Vol. 133(12), 122501 (6 pages) (2011).

[2] FERDOWS M., BANGALEE M. Z. I., CREPEAU J. C., & SEDDEEK M. A., “The effect of Variable Viscosity in double diffusion problem of MHD from a porous boundary with Internal Heat Generation”, ***Progress in Computational Fluid Dynamics,*** Vol.11(1), 54-65 (2011).

[3] ANGHEL M., TAKHAR H S.,& POP I., “Dufour and Soret effects on free convection boundary layer over a vertical surface embedded in a porous medium”, ***J. Heat and Mass Transfer,*** Vol.43, 1265-1274 (2000).

[4] POSTELNICU A., “Influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects”, ***Int. J. Heat Mass Transfer,*** Vol. 47, 1467-1472 (2004).

[5] ALAM M.S., RAHMAN M.M., “Dufour and Soret effects on mixed convection flow past a vertical porous flat plate with variable suction”, ***Nonlinear Analysis: Modelling and Control,*** Vol. 11(1), 3-12(2006).

[6] GNANESWARA REDDY., & BHASKAR REDDY., “Soret and Dufour Effects on steady MHD Free Convection Flow past a Semi-infinite Moving Vertical Plate in a Porous Medium with Viscous Dissipation”, ***Int. J. of Appl. Math and Mech.,*** Vol. 6(1), 1-12 (2010).

[7] GOVARDHAN K., BALASWAMY B., & KISHAN N., “Effects of Internal Heat Generation, Soret and Dufour on MHD Free Convection Heat and Mass Transfer in a Doubly Stratified Darcy Porous Medium with Viscous Dissipation”, ***Emirates Journal of Engineering Research,*** Vol. 17(2), 29-42 (2012).

[8] MOHAMMADEIN A.A., and El-SHAER N.A., “Influence of variable permeability on combined free and forced convection flow past a semi-infinite vertical plate in a saturated porous medium”, ***Heat Mass Transfer,***Vol.40, 341-346 (2004).

[9] NALINAKSHI N., DINESH P.A., and CHANDRASHEKHAR D.V.,“Effects of Internal Heat Generation and Variable Fluid Properties on Mixed Convection Past a Vertical Heated Plate”. **Fluid Dynamics and Material Processing (FDMP),** vol.10, no.4, pp.465-490. (2014).