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Variable Engagement Model for the Design of Fibre Reinforced Concrete Structures

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Jackie Yen Lei Voo[1] and Stephen J. Foster[2]

Abstract

In this paper a model is developed to describe the behaviour of randomly orientated discontinuous fibres in reinforced composites subject to uniaxial tension. The model is built by integrating the behaviour of single, randomly oriented, fibres over 3D space and is capable of describing the peak and post-peak response of fibre-cement-based composites in tension. The model is used to form a constitutive law for use in finite element analysis of reactive powder concrete members with a prestressed reactive powder beam failing in shear analysed. A good correlation between the theoretical and experimental results attained.

Introduction

The idea of using discrete, ductile, fibres to reinforce brittle materials such as concrete is not new with many studies having been undertaken over the past four decades. Early studies by Romualdi and Batson (1963) indicated that the tensile strength of concrete can be improved by providing suitably arranged and closely spaced wire reinforcement. The low tensile strength of concrete matrix is primarily due to the propagation of internal cracks and flaws. Romualdi and Batson hypothesized, that, if these flaws can be locally restrained from extending into the adjacent matrix, the initiation of tension cracking can be retarded and a higher tensile stength of the material achieved.

By adding fibres to a concrete mix the objective is to bridge discrete cracks providing for some control to the fracture process and increase the fracture energy. The current understanding of the behaviour of fibre-matrix interfacial mechanics is based on a number of pullout studies using single or multiple fibres where steel fibres are

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embedded within a cementitious matrix. In spite of a belief sometimes expressed (Banthia and Trottier, 1994) that no correlation exists between the behaviour of a single fibre pullout test and the behaviour of bulk fibres in a real composite matrix, the effectiveness of a fibres as a medium of stress transfer is often assessed using fibre pullout tests where slip between the fibres and the matrix is monitored as a function of the applied load.

It is well established that for quasi-brittle materials, such as concrete, loaded in tension that localization dominates the behaviour beyond the peak load and that this behaviour can be described by the load versus crack opening displacement (w) as shown in Figure 1. In this paper a deterministic model is developed for uniaxial tension based on experimental observations and probability for randomly oriented fibres in three dimensions. The model developed within is named the Variable Engagement Model.

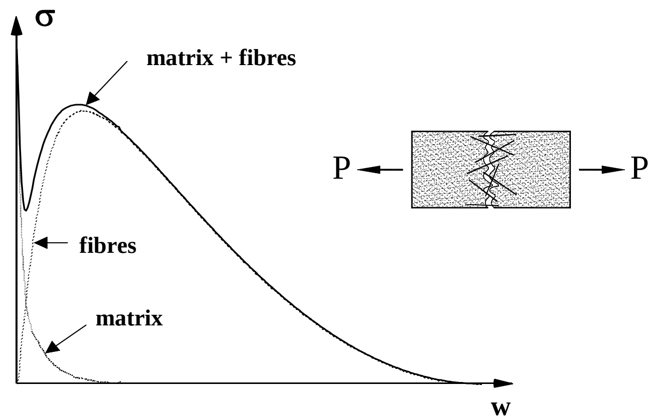


Figure 1. Stress versus COD for the matrix, fibres and the fibre matrix composite.

Variable Engagement Model

In the development of the design model that follows, the following assumptions are made: (i) behaviour of a fibre reinforced composite may be obtained by a summation of the individual components. That is, the effects of each individual fibre can be summed over the failure surface to yield the overall behaviour of the composite; (ii) the geometric centres of the fibres are uniformly distributed in space and all fibres have an equal probability of being oriented in any direction; (iii) all fibres pullout from the side of the crack with the shorter embedded length while the longer side of the fibre remains rigidly embedded in the matrix; (iv) displacements due to elastic strains in the fibres are small relative to displacements resulting from slip between the fibres and the matrix and; (v) the bending stiffness of a fibre is small and energy expended by bending of fibres can be neglected.

For mechanically anchored fibres, after the adhesion between the fibres and the matrix is broken, some slip between the matrix and the fibres must occur before the anchorage is engaged. The COD for which the fibre becomes effectively engaged in the tension carrying mechanism is termed the engagement length and denoted as w_e. Assuming the engagement-length versus fibre-slip relationship can be described using a continuous function then the boundary conditions dictate that for a fibre angle of $\theta = 0$, $w_e = 0$ and the function is to be asymptotic to $\theta = \pi/2$. One such a function is

$$w_e = \alpha \tan\theta \tag{1}$$

where w_e is the COD at the point of engagement of the fibre and α is a material parameter. To avoid variations along the plateau of the load versus COD curves in the determination of w_e, the fibre is taken to be effectively engaged at the point corresponding to 50 percent of the peak load. With this condition, the data of the single fibre tests of Banthia and Trottier (1994) for end-hooked and for crimped fibres are given in Figure 2. For the Banthia and Trottier data, Eq. 1 with $\alpha = 1.24$ for the end-hooked fibres and $\alpha = 2.31$ for the crimped fibres correlates well. The model resulting from the engagement equation (Eq. 1) is termed the variable engagement model (VEM).

For the VEM, the force in a single fibre is

$$w < w_e \quad \text{and} \quad w \geq l_a : \qquad P_f = 0 \tag{2a}$$

$$w_e \leq w < l_a : \qquad P_f = \pi\, d_f\, \tau_b (l_a - w) \tag{2b}$$

where d_f is the diameter of the fibre, l_a is the initial length of embedment of the fibre and τ_b is the mean shear stress between the fibre and the matrix measured along the remaining portion of embedded fibre $(l_a - w)$. In the analyses that follow, τ_b is taken as constant for a given fibre-matrix structure.

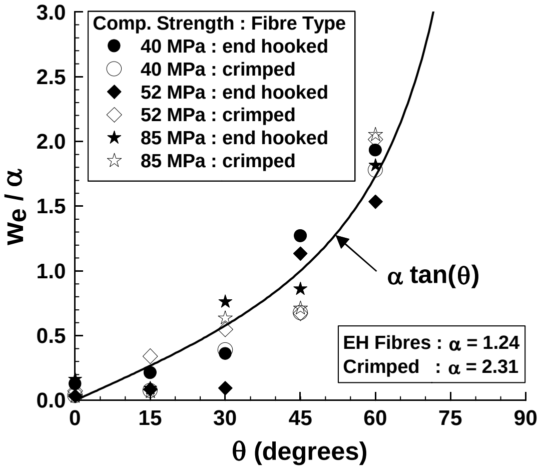


Figure 2. Engagement COD versus fibre angle for Banthia and Trottier (1994) data.

Using the concept of a fibre engagement length we can infer that for a randomly orientated fibre composite material, cracked in tension, at any point in the load-COD path there can be defined a critical angle for which fibres are becoming active. We term θ_{crit} as the point where fibres orientated at $\theta \le \theta_{crit}$ carry load while for fibres at $\theta > \theta_{crit}$ are yet to be engaged. From Eq. 1 we write

$$\theta_{crit} = \tan^{-1}(w/\alpha) \tag{3}$$

From Eq. 3 it is seen that θ_{crit} is a function of the current COD. Substituting the maximum possible fibre slip before engagement $w = l_f/2$ into Eq. 3 gives the limiting angle

$$\theta_{lim} = \tan^{-1}\left(l_f/2\alpha\right) \tag{4}$$

Whilst not all fibres at $\theta < \theta_{lim}$ may be engaged (depending of the initial embedded length l_a), no fibres at $\theta \ge \theta_{lim}$ can ever be engaged.

Stress-COD Model Excluding Fiber Fracture

For fibres randomly orientated in three dimensions, Aveston and Kelly (1973) show that the number of fibres crossing a plane of unit area is $\rho_f/2$ where ρ_f is the volumetric ratio of fibres. For fibres of length l_f and diameter d_f passing through a cracking plane with the fibre pulling out from the side with the smaller embedded length, Marti et al. (1999) noted that for $w = 0$ the average length of embedment is $l_f/4$ and that the number of bonded fibres decreases linearly with increasing COD. Rewriting Eq. 2 in the form

$$P_f = k\pi d_f \tau_b l_f/2 \tag{5}$$

gives

$$k = 0 \qquad \text{for ………} w < w_e \text{ and } w \ge l_a \tag{6a}$$
$$k = 2(l_a - w)/l_f \qquad \text{for ………} w_e \le w < l_a \tag{6b}$$

where k is denoted as the local orientation factor.

Integrating Eq. 5 over a plane of unit area one obtains the tension stress

$$\sigma = K_f \alpha_f \rho_f \tau_b \tag{7}$$

where $\alpha_f = l_f/d_f$ is the aspect ratio of the fibre, ρ_f is the volumetric fraction of fibres, τ_b is the bond stress between the fibres and the concrete and K_f is the global orientation factor.

The orientation factor can be determined by probability and is affected by the shape of the domain over which the orientation is considered. By the fibre engagement model described by Eqs. 1 and 2 one obtains

$$K_f = \frac{1}{N}\sum_{i=1}^{N} k_i = \lim_{N \to \infty} \frac{1}{N}\left\{\sum_{0}^{\theta_{crit}} k(w)\right\} \tag{8}$$

where N is the number of fibres crossing a plane of unit area and k_i is the local orientation factor for the *ith* fibre.

Given a random distribution of fibres with equal probability that any given fibre crossing a crack has a shorter embedded length of between zero and $l_f/2$, the average value of the local orientation factor for all engaged fibres is

$$k_{ave} = \frac{1}{2} - \frac{w}{l_f} \tag{9}$$

Further, if all fibre orientations have equal probability and noting that the proportion of bonded fibres decreases linearly with increasing *w*, then from Eq. 8 we write

$$K_f = \frac{2\theta_{crit}\, k_{ave}}{\pi} \bullet \left(1 - \frac{2w}{l_f}\right) \tag{10}$$

where the term in parenthesis in Eq. 10 is the proportion of fibres that have not pulled out from the matrix for a given COD. Substituting Eqs. 3 and 9 into Eq. 10 we find

$$K_f = \frac{\tan^{-1}(w/\alpha)}{\pi}\left(1 - \frac{2w}{l_f}\right)^2 \tag{11}$$

In the formulation of Eq. 11 it was assumed that all fibres are pulled out from the matrix and there is no fibre fracture. Thus, Eq. 12 applies provided that

$$l_f < l_c = \frac{d_f}{2}\frac{\sigma_{fu}}{\tau_b} \tag{12}$$

where l_c is a critical fibre length and σ_{fu} is the tensile strength of the fibre. If Eq. 12 is violated then a portion of the fibres will fracture and Eq. 11 does not apply.

Stress-COD Model Including Fiber Fracture

Assuming a constant bond shear stress along the fiber length then, by force equilibrium, any arbitrarily orientated fiber will fracture if

$$l_a \geq \frac{d_f}{4} \frac{\sigma_{fu}}{\tau_b} + w_e \tag{13}$$

For a given COD (w) the global orientation factor is given by

$$K_f = \left[\frac{2}{\pi} \cdot \frac{1}{l_f/2 - w} \int_0^{\theta_{crit}} \int_w^{l_{a,crit}} k(l_a, \theta)\, dl_a\, d\theta \right] \cdot \left(1 - \frac{2w}{l_f} \right) \tag{14}$$

where $l_{a,crit}$ is the critical fiber embedment length for fracture and is given by

$$l_{a,crit} = \min\left(l_c/2 + w_e \ , \ l_f/2 \right) \tag{15}$$

Substituting Eq. 6 into Eq. 14 gives

$$K_f = \frac{4}{\pi l_f^2} \cdot \int_0^{\theta_{crit}} \left\{ \max\left(l_{a,crit} - w \ , \ 0 \right) \right\}^2 d\theta \tag{16}$$

where θ_{crit} is given by Eq. 3. Equation 16 may be solved by numerical integration. For the case of $l_c < l_f$ no fibers fracture and Eq. 16 reduces to Eq. 11.

Comparison of VEM against Test Data for RPC

The model described by Eqs. 1 to 16 was calibrated and verified against a wide range of test data with the results given in Voo and Foster (2003) with a good theoretical to experimental correlation attained. During calibration it was found that for straight and end hooked steel fibres the engagement parameter may be taken as

$$\alpha = d_f/3.5 \tag{17}$$

Figure 3 demonstrates the VEM as given by Eqs. 7, 11, 16 and 17 compared for the tensile strength versus crack opening displacement data for a series of reactive powder concrete (RPC) fibre cocktail specimens of Rossi (1997). Three cylindrical 150 mm high by 74 mm diameter specimens (with a 15 mm circumferential notch) were tested containing 2%, by volume, of 25 mm long by 0.3 mm diameter end-hooked (EH) fibres and 5% of 5 mm long by 0.25 mm diameter straight fibres. The tensile strength of the fibres was 1200 MPa. The mean tensile strength of the RPC matrix is taken as

$$f_{ct} = 0.6\sqrt{f_{cm}} \tag{18}$$

and the fibre bond strength for RPC is taken as

$$\tau_b = 1.5\sqrt{f_{cm}} \text{ for EH fibres} \tag{19a}$$

$$\tau_b = 0.9\sqrt{f_{cm}} \text{ for straight fibres} \tag{19b}$$

where f_{cm} is the mean compressive cylinder strength. For Rossi's specimens $f_{cm} = 193$ MPa. For the EH fibres, the length of the fibres is such that a significant portion of the fibres fracture before engagement and, thus, the contribution of the EH fibres to the overall response is less significant than that of the, shorter, straight fibres.

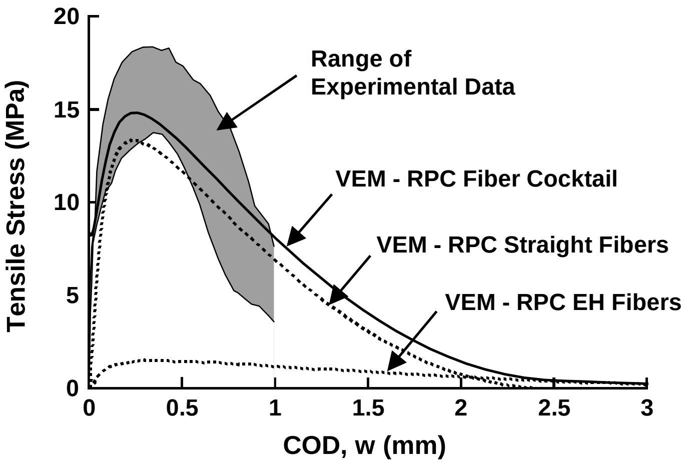


Figure 3. VEM compared with RPC fibre cocktail data of Rossi (1997).

Example - FE Modelling of RPC Girders using VEM

A series of seven prestresses RPC beams were tested by Voo et al. (2003) with the dimensions as shown in Figure 4a. A fibre cocktail beam, Beam 7, is modelled with the FE program Strand7 using the FE mesh shown in Figure 4b. The RPC was modelled using 8-node membrane elements and the prestressing steel modelled with 3-node bar elements with perfect bond assumed between the steel and the concrete. Half of the beam was analysed taking into account symmetry.

Beam 7 contained 1.88 percent, by volume, of 13 mm long by 0.2 mm diameter straight fibres and 0.62 percent of 30 mm long by 0.5 mm EH fibres. The fibre strengths for the straight and EH fibres were 1800 MPa and 1000 MPa, respectively. The mean cylinder compressive strength was 169 MPa, the tensile strength of the

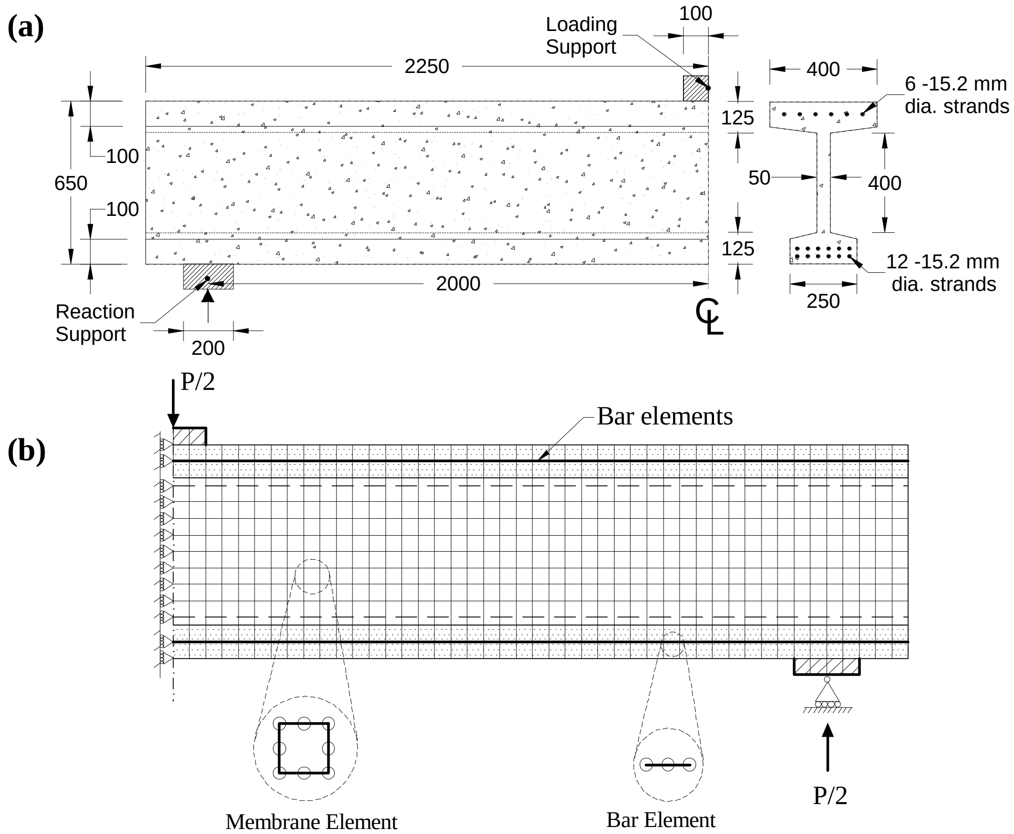


Figure 4. (a) Dimensions of RPC Beams (Voo et al., 2003); (b) FE mesh.

matrix was obtained using Eq. 18 and the fibre bond strengths calculated by Eq. 19. The initial elastic modulus and Poisson's ratio of the RPC were taken as 45 GPa and 0.15, respectively. The strands were prestressed with a force of 37.5 kN for each strand (15% of GUTS) giving initial prestressing forces of 225 kN and 450 kN for the top and bottom flanges, respectively, and an average prestress on the section of 7.2 MPa. The constitutive model for the RPC in tension was obtained using the VEM and is shown in Figure 5. The stress-strain law for tension was obtained by dividing the $\sigma - \text{COD}$ law by the characteristic length of the element and adding in the elastic component.

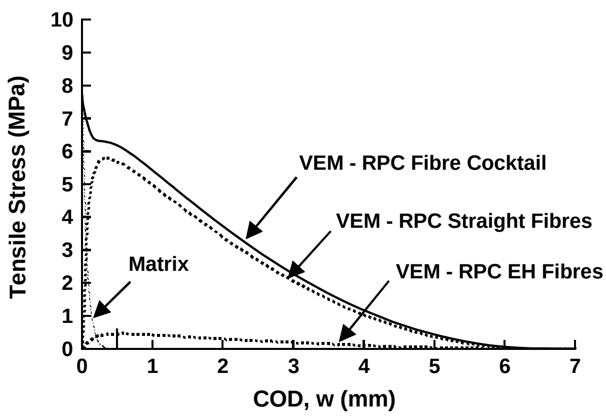


Figure 5. Tensile Stress versus COD obtained by the VEM for Beam 7.

The load versus midspan deflection output obtained from the FEM is compared with the experimental data in Figure 6. In Figure 7, the localised shear failure crack attained at the peak load for the FE analysis is compared with that of the experimental beam. It is seen from Figures 6 and 7 that the FE results compare well with the experimental observations for both the overall response of the beam and the local failure mode.

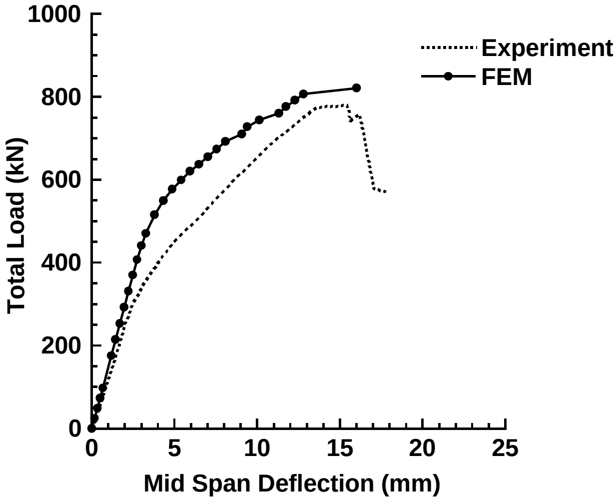
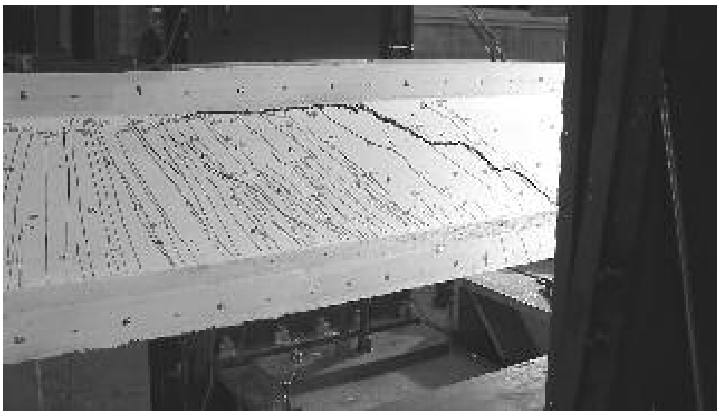


Figure 6. Load versus midspan displacement for Beam 7.

(a)



(b)

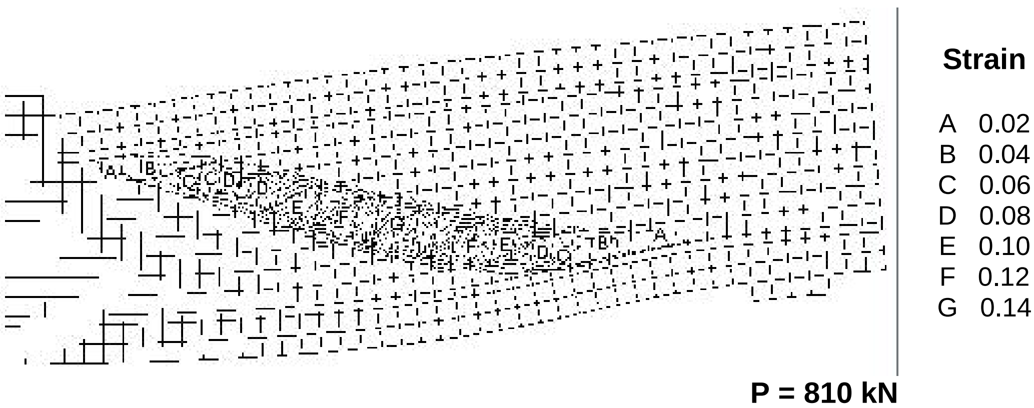


Figure 7. Shear cracking of RPC Beam 7 (a) Experiment; (b) FEM.

Conclusions

In this paper a new model for tensile stress versus crack opening displacement for mode I fracture of fibre reinforced concrete has been developed. The model was used to describe the fracture of RPC fibre cocktail with a good agreement between the experimental data and the model. Further, the VEM was used for a FE analysis of a prestressed concrete RPC beam failing in shear. The results of the FE model were shown to compare well for the overall response of the girder and for the localised shear failure mode.

Acknowledgements

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