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Maciej Matyka
Faculty of Physics and Astronomy, University of Wrocław

Zbigniew Koza
Faculty of Physics and Astronomy, University of Wrocław

Jaroslaw Golembiewski
Faculty of Physics and Astronomy, University of Wrocław

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THE VELOCITY DISTRIBUTION IN A RANDOM POROUS MEDIUM

Maciej Matyka and Jarosław Gołembiewski and Zbigniew Koza
Faculty of Physics and Astronomy, University of Wrocław, Wrocław, 50-204, Poland

ABSTRACT

We investigate the velocity distribution function of the fluid flow through a model of random porous media. We examine how the form of the distribution changes with porosity. We find a crossover porosity at which the scaling character of the distribution changes and discuss our findings in the context of a nearly exponential velocity distribution function measured recently experimentally [Datta, S.S. et al. (2013) Phys. Rev. Lett. 111: 064501].

INTRODUCTION

One of the major problems in the analysis of realistic flows through porous media is a complicated arrangement of streamlines, which follow various, tortuous pathways [1]. The problem is particularly acute for three-dimensional (3D) flows, which are often difficult even to visualize, as a projection of 3D data on a two-dimensional figure allows us to appreciate only a tiny fraction of the flow properties, as depicted in Fig. 1.

To better understand the connection between macroscopic properties of the flow and the pore-scale structure of the medium, one needs mathematical tools which are more compact and informative than the streamlines or the full velocity field itself. One such tool is the velocity distribution function (vdf) [2, 3], which describes how the local fluid velocities are distributed on the average.

The vdf describes the probability that a small, randomly chosen fluid volume has a given velocity. Unlike the famous Maxwell-Boltzmann distribution function for the ideal gas, the vdf for fluid flows reflects the randomness of the medium rather than the effect of inter-particle collisions. One might expect this effect to be strongly influenced by the porosity, and such dependence could explain the apparently conflicting results reported in previous studies. For example, Bijeljic et al. [5] computed numerically the vdfs for some real porous media using micro-CT images and found that in each case the vdfs had Gaussian shapes. Gaussian-type vdfs were also observed in a diluted, two-dimensional model of random porous media with the minimum distance constraint between grains [5]. In contrast to these studies, Datta et al. [3], who recently measured vdfs in packed glass beads, obtained a nearly exponential decay of the distribution tail.

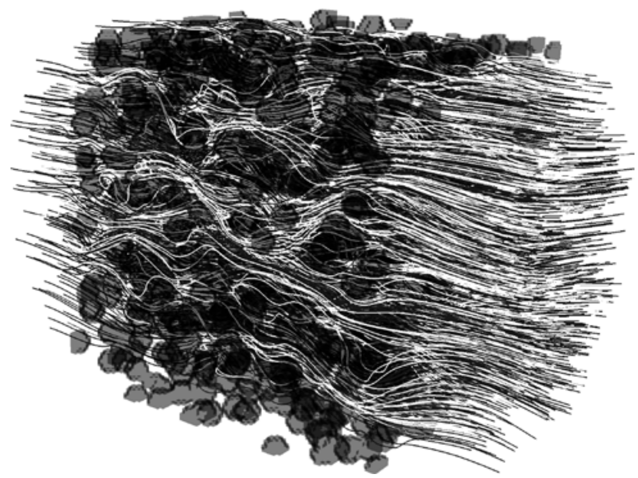

Figure 1: An exemplary fluid flow through a three-dimensional porous medium.

The goal of our work is to investigate whether the form of the velocity distribution function in a random porous medium depends on its porosity. To this end we investigate the vdf in a numerical model with controllable porosity.

1 Velocity distribution functions in model systems

Exact forms of the velocity distribution functions are known for only a couple of flow systems, which, as might be expected, are neither random nor even porous. The simplest case is the plug flow, where the velocity profile is simply a constant ($u=u_0$) [2]. In this case the vdf is given by the Dirac's delta distribution,

$$\mathrm{vdf}(u)=\delta(u_0). \quad (1)$$

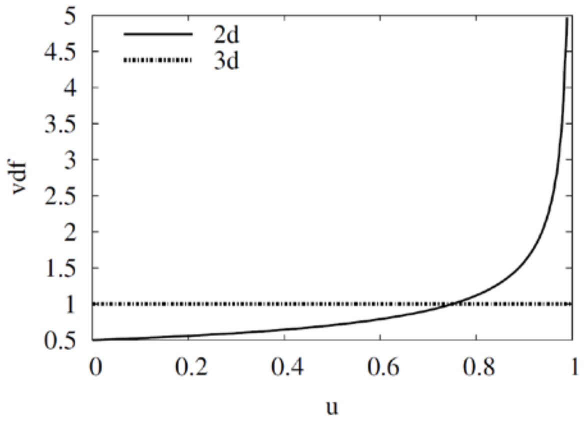

Figure 2: The velocity distribution function of the velocity parallel to the macroscopic flow direction, for the two- (Eq. 3) and three-dimensional (Eq. 2) Poiseuille flow (with u_0=1).

In the case of the Poiseuille flow, the velocity at the distance r from the pipe center is $u(r)=-u_0[1-(r/R)^2]$, where R is the pipe radius and u_0 is the maximum fluid velocity. In three dimensions the velocity distribution turns out to be flat [2],

$$\mathrm{vdf}(u)=1/u_0. \qquad (2)$$

However, in a two-dimensional Poiseuille flow the velocity takes on a different form. Following the procedure given in [2], we arrive at

$$\mathrm{vdf}(u)\mathrm{d}u=R^{-1}\mathrm{d}r= R^{-1}\,(\mathrm{d}r/\mathrm{d}u)\,\mathrm{d}u = (2u_0)^{-1}(1-u/u_0)^{-1/2}\,\mathrm{d}u, \qquad (3)$$

where $r(u)=u_0(1-u/u_0)^{1/2}$. Plots of the velocity distribution functions given by Eqs. (2) and (3) are shown in Fig. 2.

2 Random porous medium

We consider a simple model of fibrous materials (cf. [6]) with easily controllable porosity, in which one keeps on depositing at random circular obstacles (parallel cylinders) until the porosity reaches the desired value. For the sake of simplicity, we shall restrict the numerical work to the 2D space. We set up a square grid with the zero-velocity boundary conditions at the top and bottom walls, and impose the periodic boundary conditions on the remaining walls. An external bulk force (gravity) was imposed on the fluid and its value was kept low to ensure that the system is in the creeping flow regime. As we used a discrete lattice, a staircase approximation of the obstacle boundaries was used. The porosity was controlled by the volume occupied by the obstacles relative to the system volume. To solve the flow equations, we used the Palabos library, an open-source implementation of the lattice Boltzmann method (LBM), which has already been used in various computational fluid dynamics problems [7, 8], including flows through porous media [9,10].

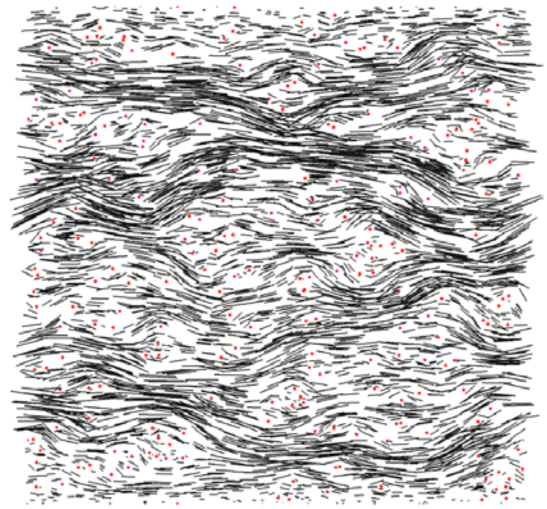

Figure 3: (Color online) Velocity vectors for φ =0.99. The system size is 1000x1000 and the diameter of each individual circular grain is a=4 lattice units.

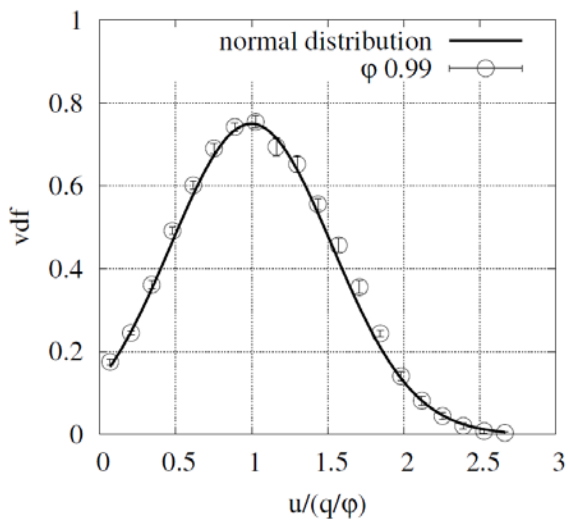

Figure 4: The velocity distribution function of the velocity parallel to the macroscopic flow direction, for the two- (Eq. 3) and three-dimensional (Eq. 2) Poiseuille flow (with u_0=1).

We used the Bhatnagar-Gross-Krook (BGK) operator for the collision term in the Boltzmann's transport equation with the numerical viscosity ν=1.0/6.0 [11, 12].

We started from determining the flow through the model at a high porosity φ=0.99 (see Fig. 3). We used an LxL lattice with L=1000 lattice units (l.u.) and impermeable disks of diameter a=4 l.u. With this parameter choice, the system is larger than the representative elementary volume [13, 14]. We then combined the results of 10 independent realizations to obtain a single velocity distribution function, which is plotted in Fig. 4.

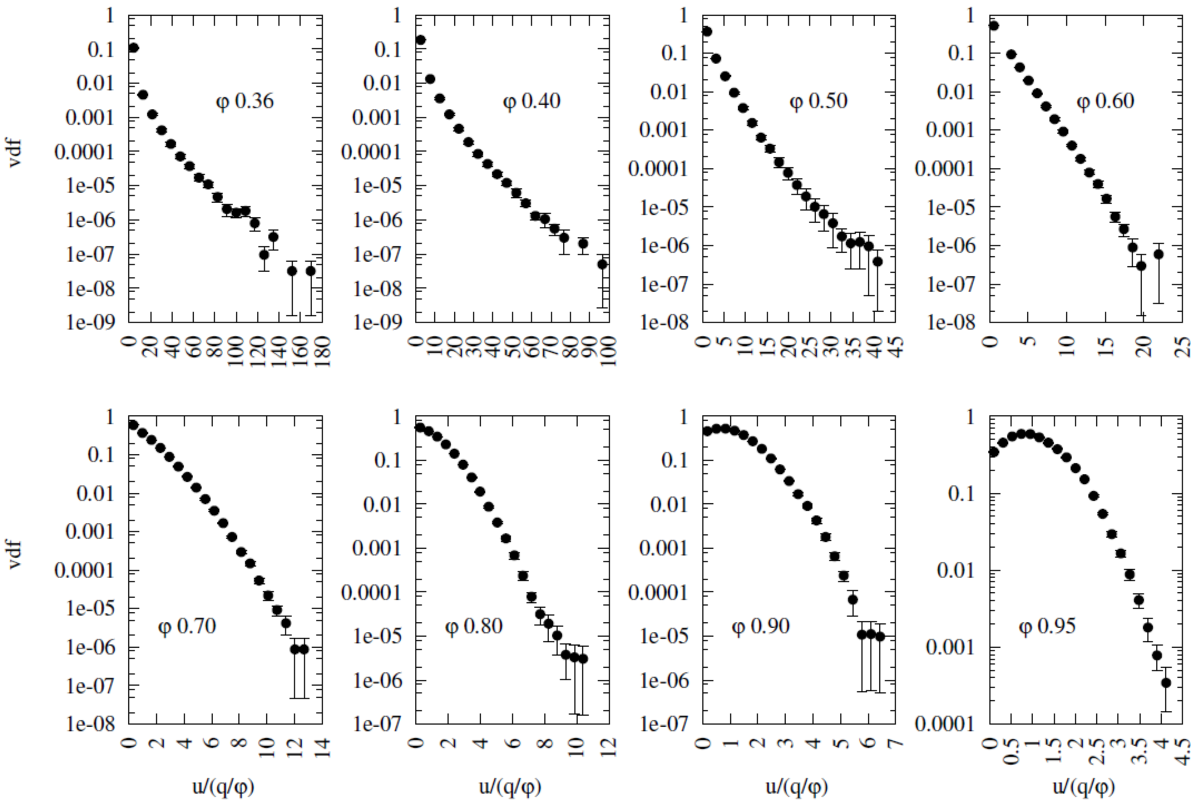

Figure 5: A semilogarithmic plot of the velocity distribution function for porosities ranging from φ =0.36 (left top) to φ=0.95 (right bottom). The error bars represent the standard error of the mean taken over 10 independent obstacle configurations at a given porosity. Note the variation of the axis scales.

The data were fitted to the Gaussian function,

$$\mathrm{pdf}(x) = \sigma^{-1}(2\pi)^{-1/2}\exp(-(x-m)^2/(2\sigma^2)) \qquad (4)$$

which yielded a convincing fit with σ=0.532 and μ=0.998.

Next we generated 10 independent random systems for each of several porosities varying from 0.95 down to 0.36. The velocity distribution functions obtained this way are depicted in Fig. 5. As can be seen, the vdf change its character significantly while going from high to low porosities. For example, at relatively high φ=0.95 the distribution can be still well approximated by the Gaussian distribution, whereas for φ=0.6 the vdf decays exponentially, and for even lower porosities the vdf has a long, subexponential tail.

To understand the origin of the normal distribution in highly porous systems, we consider a conceptual model of a two-dimensional porous medium consisting of randomly deposited, separated grains that are impermeable to the fluid (see Fig. 6). At low Reynolds number the viscous fluid penetrates the system along various pathways forming channels almost parallel to the macroscopic flow direction, but of diverse and spatially varying widths [15]. The system can be thus divided into consecutive fluid/solid segments of various widths (marked as A, B, C, D, E in Fig. 6). Solid segments are impermeable. However, in each open segment the fluid flows with approximately Poiseuille's velocity profile (cf. the profiles of the velocity at the vertical cross

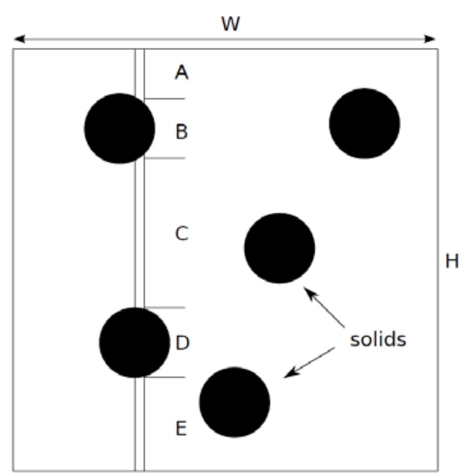

Figure 6: A simplified model of a highly porous two-dimensional system. Black circles represent solid grains. Segments A, C, and E are permeable to the flow.

sections through a porous medium shown in Fig. 4 of Ref. [15] and Fig. 5a of Ref. [1]). These profiles correspond roughly to the vdf depicted in Fig. 2. The most important feature of the vdf for an individual channel is that its first and second moments are finite. Since the maximum velocity in each segment varies with the segment width, which can be considered as a random variable, the vdf of the whole system can be regarded as an average over vdfs for individual segments. Consequently, using the central limit theorem one expects that the vdf for the whole system will be given by the normal distribution.

As the porosity is lowered, the channels become more wiggly, and there start to form dead-end regions in which the fluid is trapped and practically motionless [16]. This has a dramatic impact on the vdf and changes its form from the Gaussian through exponential to the subexponential one.

CONCLUSIONS

Our study on the velocity distribution function of a two-dimensional model of an isotropic random medium shows that for (high porosity) dilute systems it can be described by the normal distribution. A similar effect was already found for other models of random media, even at somewhat lower porosities [5]. However, the model considered in [5] had a constraint on the minimum distance between two obstacles that did not allow any touching or overlapping of obstacles, and so no dead-end zones could be formed in it.

We also found that the shape of the velocity distribution function changes as the porosity is lowered (Fig. 5). Around φ=0.60 the distribution becomes exponential (Fig. 5, right top). A similar form of the vdf was recently reported by Datta et al. [3] for an experiment on a glass bead packing at φ=0.41. The system they used was three-dimensional, which may explain a different crossover porosity obtained in our model. Anyway, we expect that this crossover porosity is connected to the clustering of the solids forming the porous matrix [17, 18] and we interpret it as the point in which separated islands start to touch each other.

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