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Scaling Relationships of Gas-Solid  
Spouted Beds

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Xu et al.: Scaling Relationships of Gas-Solid Spouted Beds

## SCALING RELATIONSHIP OF GAS-SOLID SPOUTED BEDS

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### ABSTRACT

An additional parameter, the particle-particle coefficient of restitution was added to the scaling relationship of spouted beds proposed by He et al. (4) by granular kinetic theory analysis of the particle movements in both spout and annulus regions. The experimental verification was conducted in two spouted beds with 80 and 120 mm ID. It has been found that good similarity could not be obtained when the coefficients of restitution were not matched.

### INTRODUCTION

Spouted bed technology has been widely used for a variety of applications, such as drying, coating, granulation, and chemical reactors (1). Although many equations based on small-scale spouted beds ( $D_c < 0.3$  m), most of which were operating at ambient conditions, are available for predicting the hydrodynamic properties of spouted beds, the information on large scale vessels, especially those operating at elevated temperature and pressure, is inadequate (2). Therefore, scaling-up studies of spouted beds are of great importance.

The most straightforward method to derive the correct scaling relationships is the nondimensionalization of the governing equations. The scaling relationships for spouted bed were developed based on scaling relationship proposed by Glicksman (3) for fluidized beds. By analysis of the force balance on particles in the annulus region of a spouted bed, He et al. (4) added two dimensionless parameters, the internal friction angle ( $\phi$ ) and the loose packed voidage ( $\epsilon_0$ ), to the scaling relationship of Glicksman (3). Their experimental verifications conducted in columns of different sizes at atmospheric and elevated temperatures and pressures demonstrated that the full set of modified scaling parameters are valid for spouted beds when all dimensionless parameters are matched between a prototype and a model bed. For the scaling relationship of a two-dimensional spouted bed, Costa and Tarano (5) found that the parameters of Glicksman (3) were adaptable. For the scaling relationship of a rectangular spouted bed with a draft tube, Shirvanian and Calo (6) added two parameters, the coefficient of restitution and internal friction angle, to the parameters of Glicksman (3) to account for the effect of particle-particle interactions.

However, in the scale-up study of He et al. (4), only considering the motion of the particles in annulus is inadequate, because the flow in the spout region is also significantly different from that in a fluidized bed. The flow of gas and particle phases in the spout zone is dominated by gas turbulence and particle-particle collisions. Therefore, the effective stress tensor relationships for both the spout region and the annulus region should be taken into account. It has been found that the granular kinetic theory was suitable for describing the particle-particle stresses in spouted beds, e.g. (7, 8). In the present study, a new scaling relationship of spouted beds was proposed and was experimentally verified with the data obtained in two spouted beds.

## SCALING RELATIONSHIP FOR SPOUTED BED

By nondimensionalizing the continuity and momentum equations for the fluid and solid phases along with the boundary conditions, together with the consideration of the stress tensor by using the kinetic theory of granular flow, the full set of the scaling parameters were obtained as follows. The derivation detail was not presented in this paper due to the space limitation, see (4, 7-8) for reference.

$$\left\{ \frac{gd_p}{U^2}, \frac{\rho_s d_p U}{\mu}, \frac{\rho_g}{\rho_s}, \frac{H}{d_p}, \frac{D_c}{d_p}, \phi_s, \varphi, \varepsilon_0, e_{ss}, \text{dimensionless particle size distribution, and dimensionless bed geometry} \right. \quad (1)$$

Comparing Eq. (1) with the scaling relationship of He et al. (4) shows that an additional parameter, the coefficient of restitution of particles,  $e_{ss}$ , is introduced in the present study. It is well known that  $e_{ss}$  is a measure of the elasticity of the collision between two particles, and relates to how much of the kinetic energy of the colliding particles before the collision remains after the collision. It has been shown that the flow dynamics of gas-solids systems, such as the bed expansion ratio, particle velocities were sensitive to  $e_{ss}$  (9). Huilin et al. (7) and Du et al. (8) showed that  $e_{ss}$  could greatly affect the CFD simulation results of the spouted beds. However, in their validation experiments, the particles used in different cases by He et al. (4) are same or have close  $e_{ss}$ , the possible effect of  $e_{ss}$  on the scaling of spouted bed is therefore undetectable.

## VALIDATION OF THE SCALING RELATIONSHIP

Figure 1 is a schematic of the experimental setup. The spouted beds studied were two plexiglass cylindrical columns with conical steel bases. The internal diameters of the cylindrical sections are 120 mm and 80 mm, respectively, and both are 2.0 m in height. The conical bases have an internal angle of 60°. Two gas orifices with diameters of 10 and 6.7 mm were used. The two columns are in geometrical similarity. On the top of each column, a pressure valve is installed to pressurize the system to the desired pressure. Length rules are blazed on the outside of the columns to measure the bed height and the fountain height.

Air, as spouting gas, was provided by an air compressor, controlled by a pressure regulator and measured by rotameters. Narrow size-distributed alumina and silica gel particles were used as bed materials. Their properties can be found in Table 1. The coefficients of restitution were measured by the method of Zhang and Vu-Quoc (10). In this method, a particle in a closed box was freely dropped at height  $H_0$  and

its rebounding height  $H_1$  is recorded by a high speed video camera, then the coefficient of restitution can be determined by  $e_{ss} = (H_1/H_0)^{1/2}$ . It should be noted that the close box in this measurement is not vacuum. Since additional forces, such as the drag force should be accounted for in order to get the true coefficient of restitution, the  $e_{ss}$  measured in this study can rather be served as a relative comparison of the coefficient of restitution of different particles. Bed voidage was measured by an optical fiber system.

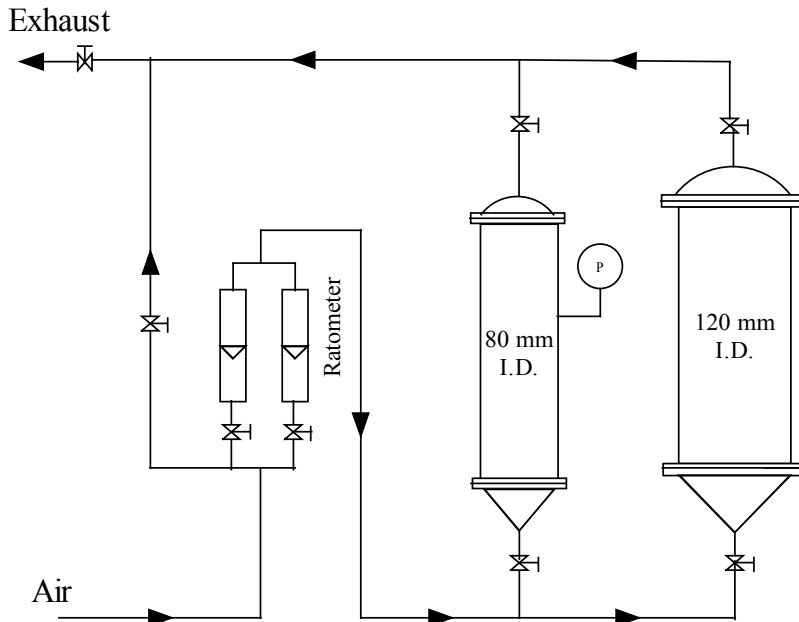


Fig. 1. Schematic of experimental setup

To verify the new scaling relationships proposed in the present investigation, several cases were designed. Test conditions are listed in Table 1, the gas is air in all the cases. Case A was conducted in the 120 mm column with 1.5 mm silica gel particles spouted at ambient conditions; Case C is conducted in the 80 mm column with 0.96 mm silica gel particles, the pressure was increased to 130 kPa to match all the scaling parameters with Case A, except for the slight difference in  $U^2/gd_p$ ; Case E is also conducted in the 80 mm column with 1.08 mm alumina particles at room temperature and 170 kPa pressure, and was designed to have the coefficient of restitution mismatch with Case A; Cases B and D are the test cases with only the coefficient of restitution mismatched with each other. Case B is conducted in the 120 mm column with 1.51 mm silica gel particles spouted at ambient conditions, while Case D is conducted in the 80 mm column with 1.08 mm alumina particles spouted at room temperature and 136 kPa. Here, Cases A and B are designed as the prototype beds and the others as the model beds for comparison. For all the cases designed, the following geometric criteria for stable spouting were satisfied (4, 12):

$$D_i / d_p < 25 - 30 \tag{2}$$

$$D_c / D_i > 3 - 12 \tag{3}$$

Table 1. Dimensions, properties and dimensionless groups in experimental tests

Case	A	C	E	B	D
$D_c$ (m)	0.12	0.08	0.08	0.12	0.08
$D_i \times 10^3$ (m)	10	6.7	6.7	10	6.7
L(m)	2	2	2	2	2
H(m)	0.25	0.158	0.179	0.25	0.179
T(K)	298	298	298	298	298
P(kPa)	101	130	170	101	136
Particle	Silica gel	Silica gel	$Al_2O_3$	Silica gel	$Al_2O_3$
$d_p \times 10^3$ (m)	1.50	0.96	1.08	1.51	1.08
$\rho_s$ (kg/m <sup>3</sup> )	1231	1557	2061	1528	2061
$\rho_g$ (kg/m <sup>3</sup> )	1.21	1.54	2.03	1.21	1.63
$\mu \times 10^5$ (Pa·s)	1.81	1.81	1.81	1.81	1.81
U(m/s)	0.368	0.392	0.307	0.448	0.445
<b>Scaling group</b>					
$\Phi_s$	1	1	1	1	1
$\alpha$ (°)	21	21	20	21	20
$\varepsilon_0$	0.57	0.59	0.54	0.58	0.54
H/ $d_p$	165.6	165.4	165.7	165.6	165.7
$D_c/d_p$	79.5	83.3	74.1	79.5	74.1
$\rho_s/\rho_g$	1017.3	1011.2	1014.8	1252.1	1264.2
$\rho_g d_p U/\mu$	37.1	32.02	37.2	45.2	43.3
$U^2/gd_p$	9.2	16.3	8.9	13.0	18.7
$\rho_s d_p U/\mu$ (10 <sup>-3</sup> )	37.79	32.38	37.75	57	54.7
$e_{ss}$	0.84	0.82	0.70	0.84	0.70

## RESULTS AND DISCUSSION

Table 2 gives the dimensionless fountain heights measured in the different cases. It can be seen that comparing with Case A, the dimensionless fountain height of Case C is 23% higher and that of Case E is 34% higher. As can be seen in Table 1,  $e_{ss}$  is the main difference in the scaling parameters among Cases A, E and C, suggesting the importance of  $e_{ss}$ . This can be further confirmed by the much larger difference in the dimensionless fountain heights of Cases B and D that have almost the same values of the scaling parameters except for their different  $e_{ss}$ .

Table 2. Fountain heights obtained in the different cases listed in Table 1

Case	Bed height (mm)	Fountain height $H_f$ (mm)	Dimensionless fountain height $H_f/D_c$	Deviation relative to Case A (%)
A	250	145	1.21	-
E	179	130	1.62	+34
C	158	120	1.50	+23
B	250	180	1.50	-
D	179	220	2.75	+83

It is known from practice that with lower values of the coefficient of restitution the particles lose a considerable amount of their momentum during the collisions. This leads to a very different bed behavior (i.e. much more clustering of particles) (13, 14). The differences in fountain height may be related to the increased tendency to clustering and thus more irregular bed behavior.

Table 3 shows the spout diameters measured in the different cases. It can be seen that the dimensionless spout diameters of Cases A and C are very close, with only a deviation of 0.4%. For Cases A and E, however, a large deviation (20.1%) emerges; the dimensionless spout diameter difference of Cases B and D is more obvious, about 53.7%. These results further emphasize that the  $e_{ss}$  significantly affect the scaling of spouted beds. By considering the force balance and solid stresses in the annulus, Bridgewater and Mathur (11) concluded that the spout diameter depended on the gas flowrate, column diameter and particle bulk density, which was further confirmed by He et al. (4). The recent CFD studies (7, 8) revealed that by affecting the solid stresses in the spout,  $e_{ss}$  could greatly impact the flow patterns in spouted beds and thus can be considered as an important parameter in describing solid stresses in the spout.

Figure 2 shows the radial voidage profiles of Cases A, C and E. In the upper section of the bed, the voidage profiles of Cases C and E distinctly deviate from that of Case A, with that of the latter more significantly lower than that of Case A; in the middle section of the bed (Fig. 2b), this deviation becomes less for both Cases C and E; in the lower section of the bed (Fig. 2c), the voidage profiles of Cases A and C approach to each other, except near the spout axis, while that of Case E shows a larger deviation from that of Case A. As seen in Table 1, the scaling parameters of Cases C and E are close to those of Case A except for the apparent difference of the  $e_{ss}$  between Case E and Cases A and C. Therefore, the large deviation of the voidage profile of Case E from that of Case A than that of Case C signifies that  $e_{ss}$  plays a very important role in the scaling relationship. The somewhat complexity of voidage distribution in the middle section may be caused by the measurement errors. Our CFD simulation work (8) has also shown that the influence of  $e_{ss}$  on bed voidage could not be neglected. The results show that a larger coefficient of restitution gives higher voidage in the spout region, which is coincident with the CFD simulation results of Huilin et al. (7). Fig. 2 also shows that in the annulus the radial porosity profiles are nearly flat, suggesting that the interaction between the wall and the particles is small, meaning that the particle-wall collision can be neglected in the scaling relationship.

Table 3. Spout diameters measured for the different cases

Case	Dimensionless spout diameter ( $D_s/D_c$ )			Average	Deviation relative to Case A (%)
	Upper section	Middle section	Lower section		
A	0.300	0.278	0.290	0.289	-
E	0.207	0.246	0.24	0.231	-20.1
C	0.293	0.282	0.282	0.286	-0.4
B	0.280	0.290	0.280	0.283	-
D	0.440	-	0.430	0.435	+53.7

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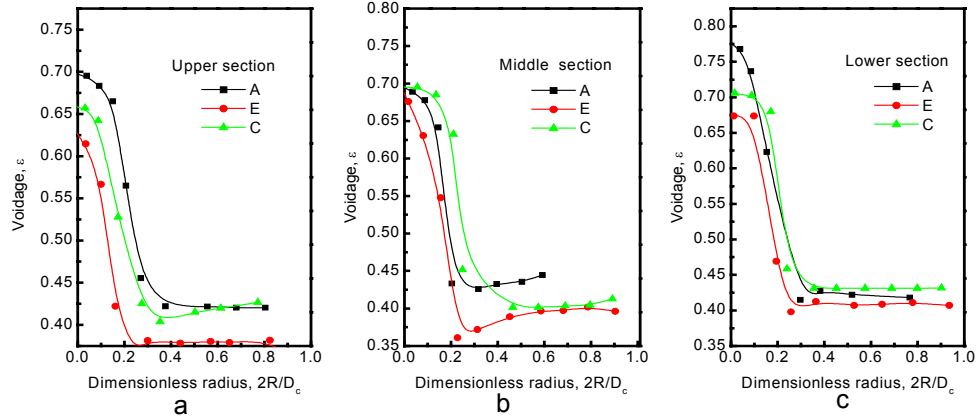


Fig. 2. Radial voidage profiles of Cases A, C and E

## CONCLUSION

A new set of scaling relationship of spouted beds including the coefficient of restitution of particles was proposed and was experimentally verified in the present investigation. The results of the experimental verification show that the hydrodynamic parameters such as the fountain height, spout diameter, and voidage profile are closely related to the coefficients of restitution. When the coefficients of restitution of two systems do not match, their similarity can not be satisfied.

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## NOTATION

$D_c$	Bed diameter, m
$D_i$	Orifice diameter, mm
$d_p$	Particle diameter, mm
$D_s$	Spout diameter, mm
$e_{ss}$	Coefficient of restitution of particles
$H$	Packing bed height, m
$H_0$	Initial height, m
$H_1$	Rebounding height, m
$H_f$	Fountain height, mm
$H_m$	Maximum spouting height, mm
$L$	Length of the vessel, m
$P$	Pressure, kPa
$r'$	Dimensionless radius

$T$	Temperature, K
$U$	Superficial gas velocity, m/s
$U'$	Dimensionless gas velocity
$\alpha$	Particle inner friction angle
$\varepsilon_0$	Solids volume fraction at loose packing state
$\mu$	Gas viscosity, Pa s
$\rho_g$	Gas density, kg m <sup>-3</sup>
$\rho_s$	Particle density, kg m <sup>-3</sup>
$\Phi_s$	Particle sphericity

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