Dynamic Characteristics of Bubbling and Turbulent Fluidization Using Hurst Analysis Technique

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ABSTRACT
A non-intrusive vibration monitoring technique was used to study the flow behavior in a fluidized bed. This technique has several advantages compared to other techniques, such as pressure probes and optical fiber probes which may influence the measurement because they are intrusive. Experiments were conducted in a 15 cm diameter by 2 m tall fluidized bed using 470 micron sand particles. Auto correlation functions, mutual information function and Hurst exponent analyses were used to analyze the fluidized bed hydrodynamics near the transition point from bubbling to turbulent fluidization regime. These methods were able to detect the regime transition point using vibration signals.

INTRODUCTION
Fluidization is a process in which solid particles become suspended and fluidized at threshold gas or liquid velocity (minimum fluidization velocity), and the bed adopts fluid-like characteristics. Today, due to their many advantages, fluidized beds have industrial applications in many areas, e.g. oil, petrochemical, mineral, biochemical, pharmaceutical, food processing. The proper functioning of fluid bed reactors requires suitable means of measuring and monitoring the bed hydrodynamics. Various techniques have been used to measure signals in fluidized beds. Pressure probes and fiber optic probes are widely used techniques but they have a shortcoming of being intrusive. This work utilized a novel non-intrusive method that measures vibrations in fluidized beds. The most common methods for characterizing time dependent signals from fluidized beds are time, frequency domain and state space analyses. Time domain approaches include observation of the time sequence of the measured signal, standard deviation analysis and analysis of other statistical moments like skewness, kurtosis and flatness (1-5).

Autocorrelation and mutual information functions are more frequently used in nonlinear state space analysis to determine time delay of reconstructed attractor (6-8). Hurst exponent analysis was developed for the first time by Hurst (9) to distinguish completely random time series from correlated time series. Hurst exponent was used by Fan et al. (10), Franca et al. (11), Drahos et al. (12), Cabrejos et al. (13), Briens et al. (14), Karamavruc and Clark (15) to assess the hydrodynamic status of the fluidized bed. Fast Fourier transform and wavelet transform are also the mathematical tools used to analyze the pressure fluctuations in fluidized bed, which express the behavior of a time series in the frequency domain.
In the present investigation, auto correlation functions, mutual information function and Hurst exponent analysis were applied to analyze vibration signals of a gas-solid fluidized bed and identify the hydrodynamics of the bed.

EXPERIMENTS

The experiment setup is schematically shown in Fig. 1. The experiments were carried out in a gas-solid fluidized bed made of Plexiglas of 15 cm inner diameter (D) and 2 m height (L). The gas distributor was a perforated plate containing 435 holes with 7 mm triangle pitch. Air was supplied by a compressor and its flow rate was measured by an orifice meter. A cyclone was placed at the column exit to return the entrained solids back to the bed. Sand particles with mean size of 470 µm and particle density of 2600 kg/m³ were used in the experiments. The system was electrically grounded to decrease electrostatics effect.

The experiments were carried out a static bed height of 22.5 cm. The same DJB accelerometer with a cutoff frequency of 25.6 kHz and sensitivity of 100 mV/ms⁻² was used to measure vibration fluctuations signals.

These measuring probes were mounted on the column 10 cm above the distributor plate by means of a magnet to minimize sudden shakes. To prevent wave interference and losing information, the sampling frequency of vibration signals was set to 25.6 kHz. All the measurements were repeated three times to ensure accuracy and reproducibility of signals.

![Figure 1. Schematic of the experimental fluidized bed set-up](image)

METHOD OF ANALYSIS

R/S Analysis

Rescaled range analysis (R/S analysis) was first introduced by Hurst (9) for studying long-term memory of a time series. Mandelbrot (16) showed that the R/S analysis is a more helpful tool in detecting long range dependence compared to more
conventional analysis like autocorrelation analysis and spectral analysis. In this method, first cumulative deviation from the mean of the time series \( x(i) \) in time window \( n \) is calculated:

\[
 x(i) = \sum_{i=n}^{N} (x(i) - \bar{x}_n)
\]

where

\[
 \bar{x}_n = \frac{1}{n} \sum_{i=1}^{n} x(i)
\]

Then, the range function \( R(n) \) is determined as maximum and minimum difference of time series \( x(i) \) in each time interval \( n \):

\[
 R(n) = \max(x(i,n)) - \min(x(i,n)) \quad 1 \leq i < n
\]

Rescaled range function is obtained by dividing \( R(n) \) by the standard deviation \( S(n) \):

\[
 \frac{R}{S} = \frac{R(n)}{S(n)}
\]

where the standard deviation \( S(n) \) is:

\[
 S(n) = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x(i) - \bar{x}_n)}
\]

It has been found that, for some time series, the dependence of \( R/S \) on the number of data points follows an empirical power law described as (10):

\[
 \left( \frac{R}{S} \right)_n \propto n^H
\]

where \( H \) is the Hurst exponent and varies between 0 and 1. The Hurst exponent is equal to 0.5 for stochastic (e.g., white noise) series, less than 0.5 for rough anti-correlated series and greater than 0.5 for positively correlated series known as persistence.

For the persistent data set, if the trend or behavior in the data set is increasing or decreasing over a certain unit interval of time, it would have a tendency to persist in increase or decrease over such an interval. Hurst exponent can be estimated by linear regression of \( \ln(R/S) \) versus \( \ln(n) \).

**Autocorrelation function**

The autocorrelation function (ACF) from the mathematics compares linear dependence of two time series separated by delay and is defined as (7-8):
\[ ACF = \frac{\sum_{i=1}^{N-\tau} [x(i) - \bar{x}][x(i + \tau) - \bar{x}]}{\sum_{i=1}^{N-\tau} [x(i) - \bar{x}]^2} \]  

(7)

where

\[ \bar{x} = \frac{1}{N} \sum_{i=1}^{N} x(i) \]  

(8)

The time delay for the attractor reconstruction is then taken at a specific threshold value of ACF where ACF is equaled to one half or zero or the first inflection point of that.

**Mutual information**

While the autocorrelation function measures the linear dependence of two variables, Fraser and Swinney (6) suggested using the mutual information \( I(\tau) \) function to determine when the values of \( x(i) \) and \( x(i+\tau) \) are independent enough of each other to be useful as coordinates in a time delay vector, but not so independent as to have no connection which each other at all. The mutual information of the attractor reconstruction co-ordinates is defined as:

\[ I(\tau) = \sum_{i=1}^{N-(d-1)\tau} P(x(i), x(i+\tau), \cdots, x(i+(d-1)\tau)) \log \left( \frac{P(x(i), x(i+\tau), \cdots, x(i+(d-1)\tau))}{P(x(i))P(x(i+\tau)) \cdots P(x(i+(d-1)\tau))} \right) \]  

(9)

where \( P(x(i)) \) refers to the individual probability of the time series variable.

In general, the time delay provided by the \( I(\tau) \) criteria is normally smaller than that calculated by the ACF(\( \tau \)) and provides appropriate characteristic time scales for the motion. As mentioned above, \( I(\tau) \) presents a kind of nonlinear correlation concept, while the ACF(\( \tau \)) provides an optimum linear correlation criterion.

**RESULTS AND DISCUSSION**

Fig. 2 shows the Hurst diagram of the vibration signal which is measured at 10 cm above the distributor for particles size of 470 µm, initial aspect ratio L/D of 1.5 and different gas velocities. According to the figure, the fluidized bed has multifractal behaviour with three different Hurst exponents. For example, consider the graph related to the gas velocity equal to 1.0 m/s, it can be seen that for small values of \( n \) (time lag), the Hurst exponent is 0.8451, which is much larger than 0.5, indicating a highly persistent dynamic feature of the fluidized bed. On the other hand, Hurst exponent is 0.494 for larger values of \( n \), which is less than 0.5, indicates a highly antipersistent dynamic feature of the fluidized bed. In general, as stated by Karamavruç and Clark (15), bubble motions correspond to higher Hurst exponents than particle motions do. It can be concluded that while Hurst exponent at small values of \( n \) or smaller fractal dimension represents the dynamic feature of macro structures, Hurst exponent at larger values of \( n \) or larger fractal dimension represents the dynamic feature of finer structures.
The Hurst exponent value of macro structures initially rises with increasing the gas velocity and then declines with further increasing of the gas velocity. This change in the trend of the Hurst exponent can be related to the regime transition of the fluidized bed. Increase in the Hurst exponent of the macro structures can be related to the growth of the bubble size due to the gas velocity increase in the bubbling regime. However, further increase in the gas velocity in the turbulent regime results in the decrease of the Hurst exponent because of the breakdown of large bubbles to voids and small bubbles. (Transition velocity from bubbling to turbulent regime was calculated equal to $U=1.23$ m/s by Bi and Grace Correlation).

As pointed out by Fan et al. (17), the reciprocal of the break point in the Hurst profile is similar to the dominant frequency of the bed. As shown in Fig. 2, the break point occurs at $n$ equal to 47 points which is equal to a time interval of about 0.00077 (s), this presents an equivalent dominant frequency of about 1396 Hz which is close to the value estimated from the power spectrum analysis of the bed vibration signal at the same conditions (9).

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![Figure 2. Hurst exponent diagram of the vibration signal measuring in tap height 10 cm above the distributor at different superficial gas velocity, particles size 470 µm, and L/D=1.5](image)

The autocorrelation function $ACF(\tau)$ and the mutual information profile $I(\tau)$ of the vibration signal which is measured at 10 cm above the distributor, for particles size of 470 µm, initial aspect ratio L/D of 1.5 and different gas velocities is illustrated in Fig. 3. For $U=1.0$ m/s, the first pass of the autocorrelation function from one half and the time delay at which the ACF becomes zero occur at 2 and 9, respectively. The first minimum of the mutual information occurs at a delay time of 26. As can be seen in this figure, these points initially occur at higher values of time delay with increasing the gas velocity, on further increase of the gas velocity results that these points occur at lower time delay. This trend can be related to the regime transition of the fluidized bed. The growth of the bubble size by increasing the gas velocity in the bubbling regime means that the behavior of the system tends to a periodic system. In periodic systems, the first pass of the
autocorrelation function from one half and the time delay at which it becomes zero, and also the first minimum of the mutual information occurs at a higher delay time in comparison with stochastic systems. When the turbulent regime is reached by the further increase of gas velocity, time delays were found to occur at lower point time delays than for the bubbling regime. This shows that the systems tends to tends to a stochastic system because of the breakdown of large bubbles to voids and small bubbles.

CONCLUSIONS

Two different Hurst exponents were identified from the vibration signals measured in the fluid bed suggesting that the fluidized bed had a multifractal behavior. Higher Hurst exponent is corresponded to macro structure in the bed, e.g. motion of large bubbles. The reciprocal of the break point in Hurst profile is similar to the main frequency of the bed. The value of the larger Hurst exponents increased with increasing gas velocity and was highest at the bubbling-to-turbulent regime transition point. The transition velocity was about 1.2 m/s, and system shows the highest periodical behavior in this point.

The autocorrelation and the mutual information functions were also used to determine the turbulent transition point from the accelerometer data. The fluidized bed system at transition point from bubbling to turbulent, the first pass of the autocorrelation function from one half and the time delay at which it becomes zero, and also the first minimum of the mutual information occurs at a higher delay time in comparison with stochastic systems, and the values of time delays were highest at the bubbling-to-turbulent transition gas velocity. These findings were similar to those of the Hurst exponent analysis.

NOTATION

ACF  autocorrelation function

\( d \)  embedding dimension
D Bed diameter
f_s sampling frequency, Hz
H Hurst exponent
I mutual information
L bed height; number of windows
N total number samples
P individual probability
R Rescaled Range function
S standard deviation
t time, s
x vibration signal (m/s²)

Greek symbols
τ Embedding time delay
τ_1 Embedding time delay related to ACF=0.5
τ_2 Embedding time delay related to ACF=0.0
τ_3 Embedding time delay related to minimum mutual information

REFERENCES