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Short crack tolerance under EAC conditions

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Short Crack Tolerance under Environmentally-Assisted Cracking Conditions

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Introduction

albeit they seem totally different, **fatigue** (which is induced by **variable loads**) and **EAC** (which can be induced by **static loads in aggressive environments**) have many similarities that can be modeled by similar **mechanical tools**

**fatigue** is a **local problem**: the **peaks** $\sigma_{\text{max}}$ and **ranges** $\Delta \sigma$ of the **stresses** acting at the **critical point** (usually a **notch tip**) drive the **initiation** of **cracks** (under nominally elastic loads), while **crack growth** is **driven** by the **ranges** $\Delta K$ and by the **peaks** $K_{\text{max}}$ of their **stress intensity factors** (**SIF**)

**EAC** is a local problem as well, driven by $\sigma_{\text{max}}$ and $K_{\text{max}}$

**SIFs** can be written as $K = \sigma \cdot \sqrt{\pi a} \cdot g(a/w) \cdot f_{\text{gr}}(K_t, a')$, where $\sigma$ is the **nominal stress**, $a$ is the **total crack length**, $a'$ is the **crack length from the notch tip**, $g$ quantifies the **cracked piece geometry** effects, while $f_{\text{gr}}$ quantifies **stress gradient effects around the notch tip**, which **control** the **behavior of short cracks** (and thus **damage tolerance**)
luckily, sharp notches with very large stress concentration factors $K_t = \sigma_{\text{max}}/\sigma_n$ are not as severe in fatigue as it could be expected from theirs $K_t \leq 1 + 2\sqrt{(b/\rho)}$ (as estimated by Inglis, where $b$ is the notch depth and $\rho$ is theirs tip radius).

indeed, for design and analysis purposes, notch effects in fatigue limits $S_L$ (meaning that stress amplitudes below $S_L$ do not initiate fatigue cracks) have long been quantified by $K_f = S_{L\text{notched}}/S_{L\text{polished}} = 1 + q \cdot (K_t - 1)$, where $0 \leq q \leq 1$ is an empirical “notch sensitivity factor”

notice that this classic concept mixes strengths (material properties) and (material-independent) stresses in the same equation, an useful (but inappropriate) trick, and also that fatigue limits $S_L$ are resistances to crack initiation, thus are associated to long (infinite?) fatigue lives

many experts still try to relate $q$ to an ill-defined “critical size distance” related to the microstructure of the material, however no such parameter has been identified so far
but **Frost** has long ago associated $q$ with the *generation* of tiny non-propagating cracks that depart from *notch tips*

thus, $q$ can be *predicted* if the *fatigue crack growth* (FCG) *behavior* of such *short cracks* is *known*

this work uses the *short crack FCG behavior* to calculate $q$ using only *mechanical properties* and sound *stress analysis* techniques, and then extends them to *EAC* problems

but how can (sharp) *cracks start* from *notch tips* and then *stop growing* if cracks obviously *much increase* the *stress concentration factor* of the *original notch*???

indeed, if notches have $K_t \cong 1 + 2\sqrt{(b/\rho)}$ but *fatigue cracks* have so *sharp tips* that their radii can be modeled as having $\rho \rightarrow 0$ ($\therefore K_t \rightarrow \infty$), then all *cracks should be unstable*

but since they *clearly are not*, cracks *cannot be modeled* by *traditional stress analysis techniques*

that is why *crack analyses* should be *based* on *SIFs*, *not* on *stresses*, *even when* the *cracks are short*
small non-propagating fatigue crack that initiated from the tip of a 1.3mm deep sharp notch with \( \rho = 70\mu m \) (\( K_t \approx 9.6 \)) very early (\( N_{ini} < 10^5 \) cycles) in the life of a low C steel specimen under a (low) \( \Delta\sigma_n = 78\text{MPa} \) nominal stress range, but then stopped with \( a_{st} < 100\mu m \) and did not grow further even after \( 2.4 \cdot 10^7 \) cycles.

Frost old data showing non-propagating fatigue cracks generated at notch tips if

\[
2S'_L/K_t < \Delta\sigma_n < 2S'_L/K_f
\]
Fatigue cracks do not initiate when \( \{ \Delta \sigma, R \} < 2S_L(R) \) and do not propagate if \( \{ \Delta K, R \} < \Delta K_{th}(R) \) (where \( R = 1 - \Delta \sigma/\sigma_{\text{max}} \) is just another way to consider their 2\textsuperscript{nd} driving force).

However, short cracks FCG thresholds \( \Delta K_{th}(a, R) \) must be smaller than long crack thresholds \( \Delta K_{th}(R) \), where \( a \) is the crack size from the notch tip (indeed, since \( \Delta K = f(\Delta \sigma \cdot \sqrt{a}) \), otherwise the stress ranges \( \Delta \sigma \) needed to grow short cracks by fatigue would be higher than the fatigue limit \( \Delta S_L(R) \) of the material, clearly a non-sense, thus short cracks must behave differently from long cracks).

It is the stress field gradient around notch tips that controls the FCG behavior of short cracks emanating from them.

\( \Delta K_{th}(a, R) \) can be modeled using a "characteristic short crack size" \( a_0 \), estimated from the material fatigue limit \( \Delta S_L(R) \) and from \( \Delta K_{th}(R) \) (not from a μstructural size).

Our short cracks are mechanically not μstructurally short, since material isotropy is assumed in their modeling.
Notch Sensitivity Predictions

- **long cracks grow** if \( \Delta K = \Delta \sigma \sqrt{\pi a} \cdot g(a/w) > \Delta K_{th}(R) \), but
- **short cracks** with \( a \approx 0 \) **cannot propagate** like them

\( \text{or else} \ \Delta K(a \rightarrow 0, R) > \Delta K_{th}(R) \Rightarrow \Delta \sigma \rightarrow \infty \), a **non-sense** since stress ranges greater than the fatigue limit \( \Delta \sigma > 2S_R \)

**can generate** and **propagate fatigue cracks** at any given \( R \)

- **to conciliate fatigue limits** (which quantify **crack initiation resistance**) \( \Delta S_0 = 2S_L(R = 0) \) with **FCG thresholds** (the long crack propagation resistance) \( \Delta K_0 = \Delta K_{th}(0) \), El Hadad, Topper and Smith (ETS) added a so-called **short crack characteristic size** \( a_0 \) to the actual crack SIF

\[
\Delta K_I = \Delta \sigma \sqrt{\pi(a + a_0)}, \quad a_0 = \frac{1}{\pi} \left( \frac{\Delta K_0}{\Delta S_0} \right)^2
\]

- **this ETS trick correctly predicts** that the largest stress range that does not propagate a microcrack is the fatigue limit: if \( a \rightarrow 0 \ll a_0 \), \( \Delta K = \Delta K_0 \Rightarrow \Delta \sigma \rightarrow \Delta S_0 \)
Kitagawa-Takahashi plot showing stress ranges $\Delta \sigma$ needed to propagate by fatigue short and long cracks under $R = 0$ in a HT80 steel with $\Delta S_0 = 575\text{MPa}$ and $\Delta K_0 = 11.2\text{MPa} \sqrt{\text{m}}$: if long cracks with $a \gg a_0$ stop when $\Delta \sigma \leq \Delta K_0/\sqrt{\pi a}$ whereas very short cracks with $a \to 0$ stop when $\Delta \sigma \leq \Delta S_0$, the ETS curve predicts (quite well) that cracks of any size should stop if $\Delta \sigma \leq \Delta K_0/\sqrt{\pi (a + a_0)}$. 
alternatively, the *short crack model* can *suppose* that the *short crack characteristic size* $a_0$ affects *FCG thresholds* $\Delta K_{th}(a)$ on (instead of the *SIFs* $K_I$), where

$$\frac{\Delta K_{th}(a)}{\Delta K_0} = \frac{\Delta \sigma \sqrt{\pi a} \cdot g(a/w)}{\Delta \sigma \sqrt{\pi (a+a_0)} \cdot g(a/w)} = \sqrt{\frac{a}{a+a_0}} \Rightarrow \Delta K_{th}(a) = \frac{\Delta K_0}{\sqrt{1+(a_0/a)}}$$

$\Delta K_0$, the *long crack FCG threshold* for $R=0$, is a *material property*, and this alternative properly *removes* the odd *SIF dependence* on *material properties* from the original *ETS model*, considering the $a_0$ *role* in the *short crack behavior* as a *modifier* of the *material FCG resistance*

$\Delta K_{thR}$, the *FCG thresholds* depend on the *second fatigue crack driving force* too, $\Delta K_{th}(a, R)$, the *short crack FCG thresholds* do as well, and this $R$-dependence (in fact this $K_{max}$-dependence) is affected by *environmental effects*

*to understand why* *short cracks* that *depart from notch tips can start* and *propagate* for a while and *then stop growing*, it must be realized that it is the *local SIF* that *drives them*
at just $b/5$ ahead of the tip of any Inglis’ hole, (surprisingly) the local to nominal stress ratio $K_{1.2} = \sigma_y(x/b = 1.2, 0)/\sigma_n \approx 2$ is almost independent of its SCF $K_t = 1 + 2b/c = 1 + 2\sqrt{(b/\rho)}$

the $K_I$ estimate for cracks that depart from the tips of an Inglis elliptical hole with $b = 10\text{mm}$ illustrates quite well how $\partial K_I/\partial a$ may decrease sharply just after the cracks initiates: $K_I \approx 1.12 \cdot \sigma_n \sqrt{\pi a} \cdot f_1(K_t, a)$, where

$$f_1 = 1 + \frac{(b^2 - 2bc)(x - \sqrt{x^2 - b^2 + c^2})(x^2 - b^2 + c^2) + bc^2(b - c)x}{(b - c)^2(x^2 - b^2 + c^2)\sqrt{x^2 - b^2 + c^2}}$$
in other words, the stress that enters in the SIF expression is the local stress that would act at the crack tip if it did not perturb the stress field, so the short crack behavior is controlled by the stress gradient ahead of the notch tip

- the sharper the notch, the higher are their SCFs $K_t$ and their gradients ahead of the crack tip $d\sigma/dx$

- cracks that depart from circular holes of radius $\rho$ in Kirsh plates have $\Delta K_I = 1.12 \varphi(x) \Delta \sigma \sqrt{\pi a}$, where $f_{gr}(K_t, x) = \varphi(x) = \left(1 + \frac{0.2}{1 + x} + \frac{0.3}{(1 + x)^6}\right) \left(2 - \frac{2.354x}{1 + x} + 1.206 \left[\frac{x}{1 + x}\right]^2 - 0.221 \left[\frac{x}{1 + x}\right]^3\right)$ is the stress gradient ahead of Kirsh holes and $x = a/\rho$

- thus, any crack departing from a Kirsh hole propagates if

$$\Delta K_I = 1.12 \varphi(a/\rho) \Delta \sigma \sqrt{\pi a} > \Delta K_{th}(a) = \Delta K_0 \left[1 + (a_0/a)^{\gamma/2}\right]^{-1/\gamma}$$

where $\gamma$ is an additional data fitting parameter

- note that $\lim_{a \to 0} \Delta K_I = 1.12 \cdot 3 \cdot \Delta \sigma \sqrt{\pi a}$, exactly as expected
\[ \Delta K_{\text{th}}(a) = \Delta K_0 \left[ 1 + \left( \frac{a_0}{a} \right)^{\gamma/2} \right]^{-1/\gamma} \]

\( \gamma = 2.0 \) generates the ETS equation, whereas \( \gamma \to \infty \) leads to the bi-linear (in log-log) estimate: \( \Delta \sigma(a \leq a_0) = \Delta S_0 \) for short cracks and \( \Delta K_{\text{th}}(a \geq a_0) = \Delta K_0 \) for long cracks.

The data-fitting parameter \( \gamma \) allows the \( \Delta K_{\text{th}}(a) \) estimates to better correlate with experimental short crack data, since most such data is bounded by \( \gamma = 1.5 \) and \( \gamma = 8.0 \).
$\frac{\Delta K_{th}(a)}{\Delta K_0}$ data, showing the ratio between the short and the long crack propagation thresholds

$\frac{\Delta K_{th}(a)}{\Delta K_0} = \left[1 + \left(\frac{a_0}{a}\right)^{\gamma/2}\right]^{-1/\gamma}$

where $\gamma=1.5$, $\gamma=2$, and $\gamma=8$.
"notch sensitivity" $q$ vs. the Kirsh hole radius $\rho$, estimated using mean $\Delta K_0$, $\Delta S_0$ and $S_U$ from 450 steels and Al alloys for $\gamma = 6$, reproduces quite well old Peterson’s data.
the SIF range for a crack \( a \) that starts in \textit{elliptical notches} with \textit{semi-axes} \( b \) \( e \) \( c \) (crack \( a \) aligned to \( b \)) in mode I is

\[
\Delta K = F \left[ \frac{a}{b}, \frac{c}{b} \right] \cdot \Delta \sigma \sqrt{\pi a}, \quad F \equiv f(K_t, s) = K_t \sqrt{\frac{1 - \exp(-K_t^2 \cdot s)}{K_t^2 \cdot s}}
\]

\[
K_t = \left[ 1 + 2 \frac{b}{c} \right] \cdot \left[ 1 + \frac{0.1215}{(1+c/b)^{2.5}} \right], \quad s = \frac{a}{a + b}
\]

\( \text{notch sensitivity} \)

\( q \) \text{ versus the semi-elliptical notch radius} \( \rho = \frac{c^2}{b} \) \text{ for typical Al alloys} \((\Delta S_0 = 129\text{MPa}, \Delta K_0 = 2.9\text{MPa} \sqrt{\text{m}}, \gamma = 6)\) \text{ loaded in mode I depends also on} \( c/b \)
this simple 2D model can be expanded to 3D to consider \textit{thickness effects around the notch tip}, using fancy \textit{FE} techniques (localized \textit{EP effects} can be considered as well, but they require even fancier \textit{FE} models)

there is no point in detailing such advanced tools here, but it is worth to present a general purpose \textit{2D estimate} for the short crack tolerance in notched DC(T) of a 6351 T6 Al, with $S_Y = 285$, $S_U = 317\text{MPa}$, $\Delta K_{th0} = 4\text{MPa} \sqrt{\text{m}}$ (properties measured in standard ASTM tests) and an estimated fatigue limit $S_L(R = -1) = 103\text{MPa}$, which by \textit{Goodman} gives $\Delta S_{L0} = 2S_LS_U/(S_L + S_U) = 155\text{MPa}$ and thus $a_0 = 211\mu\text{m}$

the idea is to use \textit{Creager} \& \textit{Paris} to estimate the effects of the notch \textit{SCF} $K_t$ and of its gradient ahead of the notch tip in the short crack \textit{SIF}, to \textit{predict} the \textit{stress ranges} that can \textit{initiate a crack} but not \textit{propagate} it until fracture
Estimate by Creager & Paris for the short crack behavior in notched DC(T)s with width \( w \), crack with size \( a \) departs from an elongated notch with depth \( b \) and tip radius \( \rho \).

DC(T) thickness \( t \), initial ligament \( l_g = w - b \), relative crack size \( a_w = (a + b)/w > 0.2 \), load \( P \).

\[
K_1 = \frac{P}{t \sqrt{w}} \left[ \frac{2 + a_w}{(1 - a_w)^{1.5}} \right] \left[ 0.76 + 4.8 \cdot a_w - 11.58 \cdot a_w^2 + 11.43 \cdot a_w^3 - 4.08 \cdot a_w^4 \right]
\]

\[
K_t = \frac{2K_1}{(\sigma_a \sqrt{\pi a})} \quad \sigma_a = \sigma_{an} + \sigma_{inl} = P/(l_g \cdot t) + 6P(b + l_g/2)/(t \cdot l_g^2)
\]

Material properties: \( S_U = 317 \quad S_Y = 285 \quad S_L = 103 \quad \Delta K_{th0} = 4 \)

\[
\Delta S = \frac{S_L - S_U}{S_L + S_U} \quad a_0 = \left( \frac{1}{1/2} \right)^2 \Delta S L_0 = 153.481 \quad a_0 = 2.107 \times 10^{-4}
\]

To initiate a crack from the notch tip at \( R = 0 \), \( P = \Delta P > \Delta S L_0 \), and to stop this crack it must reach \( K = \Delta K < \Delta K_{th0} \).

Notched DC(T) properties: \( w = 0.03644 \quad b = 0.015 \quad \rho = 0.0035 \quad t = 0.01 \quad P = 1500 \quad \rho = 0 \)

\[
a_w(a, b) = \frac{a + b}{w} \quad ax(a, b) = 0.265 \quad lg(a, b) = w - (b + a) \quad lg(a, b) = 0.041
\]

\[
faw(a, b) = \left[ \frac{2 + a_w(a, b)}{(1 - a_w(a, b))^{1.5}} \right] \left[ 0.76 + 4.8 \cdot a_w(a, b) - 11.58 \cdot a_w(a, b)^2 + 11.43 \cdot a_w(a, b)^3 - 4.08 \cdot a_w(a, b)^4 \right] \quad faw(a, b) = 5.083
\]

\[
KI(a, b, P) = \left( \frac{P}{t \sqrt{w}} \right) faw(a, b) \quad KI(a, b, P) = 3.211 \times 10^6
\]

\[
on(a, b, P) = \left( \frac{P}{l_g(a, b) + t} \right) + \frac{6P(b + 0.5 \cdot lg(a, b))}{t \cdot lg(a, b)^2} \quad on(a, b, P) = 2.234 \times 10^7
\]

\[
K_f(a, b, P, \rho) = 2KI(a, b, P) \quad K_f(a, b, P, \rho) = 7.252 \quad onm(a, b, P, \rho) = Ki(a, b, P, \rho) \quad onm(a, b, P, \rho) = 1.62 \times 10^8
\]
\begin{align*}
K_l(a,b,P,p) &= \frac{2K_l(a,b,P)}{\text{sn}(a,b,P)\sqrt{\kappa}} \\
K_l(a,b,P,p) &= 7.252 \\
\text{omax}(a,b,P,p) &= K_l(a,b,P,p)\text{sn}(a,b,P) \\
\text{omax}(a,b,P,p) &= 1.62 \times 10^9
\end{align*}

\begin{align*}
g_l(a,b,P,p) &= \frac{K_l(a,b,P)}{\text{sn}(a,b,P)\sqrt{2\pi \left( a + \frac{b}{2} \right)}} \\
K_l(a,b,P,p) &= K_l(a,b,P)g_l(a,b,P,p) \\
K_l(a,b,P,p) &= 1.164 \times 10^7
\end{align*}

\[a = 0.00001, 0.01\]

\[\Delta K_l(a) = \frac{\Delta K_l(a)}{\sqrt{1 + \frac{a}{b}}}\]
notched DC(T) being fatigue tested at the lab with a strain gage at the back face of the specimen to detect the short crack initiation within a 20μm resolution since the root radius to thickness ratio ρ/t = 0.05 is small in this test, the crack initiates inside the notch and must grow for a while with a 2D crack front before cutting the faces of the specimen this simple 2D model estimates are within 10% of FE calculations
3D Effects Around Notch Tips

3D mesh close to the notch tip, for the elliptical hole with \( b/a = 0.5 \) and \( \rho/a = 0.25 \)
3D notched plate under uniaxial load with Cartesian coordinate axes origin at the center of the notch tip.
$K_{\sigma}/K_t$ distribution along the notch front, for an elliptical hole with $b/a = 0.5$ and $\rho/a = 0.25$.
$K_{σ_{max}}/K_t$ and $K_{ε_{max}}/K_t$ variation with the thickness to root radius ratio $B/ρ$ for elliptical holes.
variation of \( K_{\sigma_{\text{max}}}/K_t \), \( K_{\sigma_{\text{mp}}}/K_t \), and \( K_{\sigma_{\text{surf}}}/K_t \) with the thickness to root radius ratio \( B/\rho \) for the elliptical holes.
Short Cracks Behavior under EP Conditions

Under contained EP conditions around crack tips, which invalidate the use of SIFs to quantify local crack driving forces, the non-propagating short crack problem can be modeled using the J-integral approach.

Like in the LE case short fatigue cracks must have higher FCG rates than long cracks in the EP case as well.

It is convenient to modify their $J_{th}(a)$ FCG threshold to consider short crack characteristic size $a_0$ effects to account for their peculiar behavior near EP notch tips.

In the LE case, the size-dependent threshold $J_{th}(a)$ must be given by $K_{th}(a)/E'$, where $E' = E$ or $E' = E/(1 - v^2)$ for plane stress or plane strain limit conditions.

In this way, $J_{th}(a)$ can then be easily compared with the crack driving force quantified by $J$ when modeling the EP short crack behavior.
if the stresses controlled by $J$ grow proportionally to the load $P$ applied on the cracked piece, then for a Ramberg-Osgood material with strain-hardening coefficient $H$ and exponent $h$, it can be shown that the crack driving force $J$ is given in clear engineering notation by

$$J = J_{el} + J_{pl} = \frac{K_I^2}{E'} + \left[ \left( \frac{P}{P_{pc}} \right) S_Y \right]^{\frac{1}{1+h}} \left[ \frac{w-a}{H^{1/h}} \right] h(a/w,h)$$

*K_I(P)* is the SIF that would be applied on the cracked piece if it remained LE, $P_{pc}$ is the plastic collapse load, $S_Y$ is the yielding strength, $w$ is the cracked piece width, $w-a$ is its residual ligament, and $h$ is a non-dimensional function that depends on the cracked piece geometry and on the strain-hardening exponent

- although not as easy to find as $K_I$ values, $h$-values may be found in tables for some simple geometries (but nowadays they can be calculated in most FE codes)
to model the short crack behavior, like its LE analog $K_{th}(a)$, the size-dependent EP (short) crack propagation threshold $J_{th}(a)$ must include the $a_0$ effect $J_{th}(a) = J_{th}/(1 + a_0/a)$

hence, like in the LE case, EP cracks grow whenever their driving force $J$ is higher than their size-dependent threshold $J_{th}(a)$, a well defined material property both in fatigue and EAC, and short cracks that depart from EP notch tips can stop when their gradient-affected driving force $J(a) = J_{th}(a)$

cracks that depart from notch tips can be much affected by the notch stress gradient when their size is small or similar to the notch tip radius $\rho$, so they can start and then stop after growing for a while, becoming thus non-propagating

this purely mechanical explanation can be applied both to fatigue and to EAC problems

we are now working on a $f_{gr} K$-modifier that considers EP effects around notch tips to avoid the need to use $J$ in crack tolerance predictions
The Measurement of Fatigue Limits

**characteristic short crack sizes** $a_0$ depend on fatigue limits, which are difficult to measure by traditional methods, but **thermographic techniques** have been recently proposed to obtain fatigue limits in a much cheaper and fast way.

They can be much more efficient than the standard up-and-down Dixon’s sequential method, which starts by testing a coupon under a given stress amplitude and if it breaks the next coupon stress level is decreased by a pre-defined stress increment, whereas if it does not break, the next coupon stress level is increased by the same increment.

**Thermography**, on the other hand, just needs to test a few specimens that do not even need to be loaded until failure. It measures the **heat generated** on the **fatigue specimen surface** by the **stress range applied on it**, to find abrupt changes in temperature induced by the transition from elastic to cyclic plastic strains, the cause for fatigue damage.
• typical $dT/dN_1 \times \sigma_a$ plot, with its bilinear trend
25 specimens tested for the Dixon’s up-and-down approach, with a load increment or decrement ratio $\Delta \sigma = \sigma_a / S_U = 2\%$, with the fatigue limit defined as survival after $5 \cdot 10^6$ cycles, $S_L = 308.9 \pm 7.1\text{MPa}$

The average fatigue limit obtained by testing 5 specimens using thermography procedures was $S_L = 305.8 \pm 5.3\text{MPa}$, a value just \textit{1.01\%} smaller than the fatigue limit obtained by the traditional Dixon’s method.
Tolerable Short Crack Sizes

The methodology presented here can be used to propose a clear and unambiguous *tolerance criterion* for small crack-like defects, a quite useful tool for practical applications.

Large cracks may be easily detected and dealt with, but small cracks may pass unnoticed even in careful inspections.

In fact, if they are smaller than the detection threshold of the NDI used to find them, they simply cannot be detected.

Thus, *structural components designed for very long fatigue lives should be tolerant to such short cracks*.

However, this most sensible and self-evident requirement is still not usually included in fatigue design routines, since practical long-life designs just intend to maintain the stress range at critical points below their fatigue limits at a given 

\[ R = \sigma_{\text{min}} / \sigma_{\text{max}} \]

ratio, \( \Delta \sigma < S_L(R) / \phi_F \), where \( \phi_F \) is a suitable safety factor against fatigue failures.
nevertheless, most long-life designs work quite well, thus they are somehow tolerant to undetectable or functionally admissible short cracks

but the question “how much tolerant” cannot be answered by SN or εN procedures alone

such problem can be avoided by adding to the “infinite life” design routine a criterion to tolerate a small crack of size \( a \), which in its simplest version should then be written as

\[
\Delta \sigma < \Delta K_R \left/ \left\{ \sqrt{\pi a \cdot g(a/w)} \cdot \left[ 1 + (a_R/a)^\gamma/2 \right]^{1/\gamma} \right\} \right.
\]

\[
a_R = (1/\pi) \cdot [\Delta K_R / \eta \Delta S_R]^2
\]

fatigue limits \( \Delta S_R \) implicitly consider effects of structural defects inherent to the material, thus this equation includes and complements them considering the tolerance to short cracks of the structural component, but regrettably (or not) there is no time to detail this mechanics here, but a simple case study can clarify how useful this concept can be
Practical Example

Due to a rare manufacturing problem, a batch of an important component was marketed with small surface cracks, causing some unexpected field failures.

The task was to estimate the largest small crack those steel components could tolerate under uniaxial fatigue loads.

Their rectangular cross section had 2 by 3.4mm and their properties were $S_L(R = -1) = 246\text{MPa}$ and $S_U = 990\text{MPa}$.

So its fatigue strength at any $R$ is estimated by Goodman (e.g.) using

$$S_R = \left[ S_L S_U (1 - R) \right] / \left[ S_U (1 - R) + S_L (1 + R) \right]$$

The mode I stress range $\Delta\sigma$ tolerable by this piece when it has a uniaxial surface crack of depth $a$ is then given by

$$\Delta\sigma < \frac{\Delta K_R / \varphi_F}{\sqrt{\pi a \cdot g \cdot [1 + (a_R/a)^{\gamma/2}]^{1/\gamma}}} \cdot a_R = (1/\pi)(\Delta K_R / \eta \Delta S_R)^2$$

$$g = \left[ 0.752 + 2.02 \frac{a}{w} + 0.37 \left[ 1 - \sin \frac{\pi a}{2w} \right]^3 \right] \cdot \sec \frac{\pi a}{2w} \sqrt{\frac{2w}{\pi a} \tan \frac{\pi a}{2w}}$$
note how small cracks with $a < 30 \mu m$ have practically no effect in this component fatigue resistance
if the cracks behave well under EAC conditions, then a Kitagawa-like diagram can be used to quantify tolerable stresses, using the material EAC resistances to define a short crack characteristic size $a_0 = \frac{1}{\pi} \left( \frac{K_{IEAC}}{\eta \cdot S_{EAC}} \right)^2$.
this means that if cracks under EAC conditions behave as they should, meaning that their \textit{driving force} is the \textit{stress intensity factor} applied on them, whereas the \textit{chemical effects} can be included in the \textit{material resistance to crack initiation} in smooth surfaces quantified by $S_{EAC}$, and its \textit{resistance to crack propagation} measured by $K_{IEAC}$, then it can be expected that:

- like \textit{fatigue cracks}, \textit{cracks} induced by EAC (under static, not dynamic loads) may \textit{depart} from sharp notches and \textit{then stop}, due to the stress gradient ahead of the notch tip, \textit{becoming non-propagating cracks}

- the \textit{size} of such \textit{non-propagating} (short) \textit{cracks} can be \textit{calculated} by procedures similar to the fatigue case

- $S_{EAC}$ \textit{cannot be measured} in notched TS considering only their $K_t$ \textit{effect}, since their \textit{gradient} is also \textit{important}

- but the resistance of notched \textit{components} to EAC can be \textit{properly quantified} by their \textit{notch sensitivity factors $q_{EAC}$}
indeed, the *structural design criterion* to avoid *EAC* problems in notched structural components should be:

\[
\sigma_{\text{max}} \leq \frac{1}{\pi} \left[ \frac{K_{\text{IEAC}} (1 + a_0/a)^{\gamma/2}}{\sqrt{\pi a} \cdot g(a/w)} \right]^{-1/\gamma}
\]

notice that this equation can indeed be used for *structural design purposes*, thus it can possibly substitute the present *pass/non-pass criterion* still used to “solve” most practical *EAC* problems nowadays.

- even though economically questionable, a *pass/non-pass criterion* may be *OK* for *design purposes*, but it is *useless* for *analysis purposes* when operational conditions change.
- the proposed *criterion* uses a *purely mechanical* approach.
- so it can be applied by structural engineers, since it does *not require* much *expertise* in *chemistry* to be useful.
- moreover, it *can be properly tested* and become a really useful engineering tool.
Verification of SCC Predictions

- EAC data measured testing the annealed Al 2024 – Ga pair (quick EAC reactions, non-toxic), following standard procedures

\[ S_{EAC} = 43.6 \pm 4.2 \text{ MPa (9 samples, 95\% reliability)} \]
\[ K_{IEAC} = 8.8 \pm 0.3 \text{ MPa}\sqrt{\text{m (8 samples, 95\% reliability)}} \]

- then 4 pairs of C(T)-like test specimens designed to support
\[ \sigma = 90\text{Mpa} > 2S_{EAC} \text{ at their notch tips} \]
\[ \{a, r, a/w\} = \{20, 0.5, 0.33\}, \{12, 0.5, 0.2\}, \{20, 0.2, 0.33\}, \{40, 0.45, 0.67\} \]
- the idea was, of course, to play with the SCF/gradient combination in order to assure the tolerance to short cracks that should start at the tips of the notches, since they were loaded well above \( S_{EAC} \)

- all the 8 test specimens started cracks at the notch tips, but NONE of them failed, exactly as predicted before the tests!
2024 T6 Al 1000x500x12.7mm plate, annealed to remove residual stresses, $E = 70$ GPa, $S_Y = 113$ MPa, $S_U = 240$ MPa, $\varepsilon_U = 16\%$, TS machined in TL direction

- Ga applied at about $35^\circ$C with a brush, using light bulbs to maintain the temperature
- Sensitivity to EAC tested at $10^{-5}$ mm/s in an Instron 5582
- $S_{EAC}$ and $K_{IEAC}$ measured in proof rings using load steps following standard procedures
\textbf{S}_{\text{EAC}} \text{ tests: initial load } 30\text{MPa, load steps } 2.5\text{MPa, 1h interval, } S_{\text{EAC}}(95\%) = 43.6 \pm 4.2\text{MPa (9 specimens)}

\textbf{K}_{\text{IEAC}} \text{ tests: } a_0/w = 0.25 \text{ pre-crack, initial load } 7.5\text{MPa}m^{0.5} + 0.25\text{MPa}m^{0.5} \text{ steps, 1h interval, } K_{\text{IEAC}}(95\%) = 8.79 \pm 0.27 \text{MPa}m^{0.5} (8 \text{TS})
notched TS designed to sustain 90MPa at their tips, about twice their SEAC

notch tip radii carefully machined and verified

four different pairs of TS, with \( \{a, r, a/w\} = \{20, 0.5, 0.33\}, \{12, 0.5, 0.2\}, \{20, 0.2, 0.33\}, \{40, 0.45, 0.67\} \), a and r in mm, all tested under constant load
Exactly as predicted before the tests, all the 8 notched test specimens initiated at least one crack under a load at the notch tip twice larger than $S_{EAC}$, but none of them failed in spite of the load maintained for a time at least 20 times longer than the time used to measure $K_{IEAC}$.
Further tests are being made in a **S13Cr super chromium martensitic stainless steel** with $S_Y = 838$, $S_U = 861$MPa, $HRc = 26$, inside a deaerated **NaCl** aqueous solution with added **H$_2$S** and **CO$_2$** and absence of **O$_2$**, at **25°C** and **pH** control with **HCl** as the aggressive environment.

$S_{EAC} = 469$MPa was measured following **ASTM F1624 step-loading procedures** in 5 specimens.
KIEAC = 36.9MPa\(\sqrt{m}\) was measured following NACE TM 0177-05 procedures in 3 wedge-loaded DCB specimens pre-cracked by fatigue.

The short crack tolerance predictions were tested in notched C(T)-like specimens, similar to those used before for the Al-Ga pair.

A 2D FE model was used to calculate the SIFs for the short cracks that depart from the notch tip, a necessary step to make the short crack tolerance predictions, and to verify their estimates based on the Creager & Paris approximation.
notched C(T)s being tested inside the aggressive environment
non-propagating short crack initiated at the notch tip under a local stress $\sigma = 0.95S_Y > S_{EAC}$
Conclusions

- clear and well defined mechanical tools have been used to model and to predict the behavior of short cracks that depart from notch tips, both under fatigue and EAC conditions.
- such short cracks can start and grow for a while, and then stop becoming non-propagating.
- in such cases the cracks can thus be considered as tolerable defects both for structural design and for structural integrity evaluation purposes.
- predictions based on those mechanical tools were verified in properly designed fatigue and EAC tests.
- even though the actual behavior of short cracks may involve non-trivial 3D and EP issues, elaborated FE models show that relatively simple estimates based on well-known 2D tools can be used within a relatively small uncertainty to generate workable engineering predictions.
more details in the 2016 *English* edition of the *FATIGUE* books by *Castro & Meggiolaro*, available at Amazon (Europe and US)

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