7-5-2016

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Proceedings of the 6th International Conference on Porous Media and Its Applications in Science and Engineering
ICPM6
July 3-8, 2016, Waikoloa, Hawaii

FRACTIONAL TRANSPORT MODELS FOR SHALE GAS IN TIGHT POROUS MEDIA

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ABSTRACT

A nonlinear transport model for single-phase gas through tight rocks, is combined with a fractional calculus method, to produce a new time-fractional advection-diffusion transport model for the pressure field, $p = p(x,t)$ in the flow of gas through tight porous reservoirs. Solutions for different fractional order, $0 < \propto < 1$, and for different nonlinear models with different apparent diffusivity K and apparent velocity U are compared. These systems could represent the gas transport in porous media where the representative control volumes are small but not infinitesimal. Applications are possible in many areas, such to shale gas recovery, and also aquifers.

INTRODUCTION

Modelling the flow of fluid through tight porous media, such as unconventional hydrocarbon reservoirs, is very challenging [1,2,3]. It is a growing sector and urgently needs to be addressed. Shale gas is found in tight porous rocks which are characterized by nanoscale porous networks with ultra-low permeability [4]. A small volume of rock contains heterogeneous structure – mostly a pore network of high complexity, and solid rock material. A conventional infinitesimal approach to balance equation is likely therefore to be of limited use; but a fractional approach which can model, at least in principle, such heterogeneity at the small but finite scales may offer a significant advantage is modeling such transport systems [5].

Conventional transport models incorporating varying degrees of realism have been proposed [6,7,8], but there still remains a significant gap between model predictions and the actual data. However, significant progress has been made recently in a new transport model proposed by Ali & Malik [1,2,3], which incorporates a greater degree of realism than previous model. A key aspect of their model is to retain all model parameters to be fully pressure dependent at all times. The model also incorporates multiple flow regimes, such as Knudsen diffusion, and also includes a nonlinear term for fast flowing gas regions in the porous media. The model was demonstrated to predict rock properties in shale rock core samples much more accurately than any previous models.

Here, we take a step further and explore nonlinear fractional transport models for single-phase gas in homogeneous tight rocks [1] which combines the conventional transport models with a fractional calculus method. We thus pose a time-fractional advection-diffusion transport model [1,9,10] for the pressure field, $p(x,t)$;

$$\frac{\partial^{\propto} p}{\partial t^{\propto}} = \frac{\partial}{\partial x}\left(-Up + K\frac{\partial p}{\partial x}\right) + R(x,t), \qquad t > 0, \quad a \le x \le b; \quad 0 < \propto < 1, \quad (1)$$

with suitable initial and boundary conditions (see below), where $\frac{\partial^{\propto}}{\partial t^{\propto}}$ is the Caputo fractional derivative or order $\propto$, R is a source term, and $b - a$ is the length of the reservoir (which could be infinite). In these models, the apparent diffusivity is $K = K(p)$, and the apparent convective velocity is $U = U(p, p_x)$; thus either or both of K and U can be nonlinear.

The model in equation (1) is the fractional version of the new model proposed by Ali & Malik [1,3]. It is important to note that fractional models have been shown to conserve properties, such as mass [11], for optimal choice of fractional order $\propto$ for a given system.

The aim here is to explore the solutions of the pressure field over a period of time in a 1-dimensional domain, for different types of non-linear fractional transport models, and for different fractional orders. A key question is to compare the solutions between them, and also with the conventional cases, i.e. with $\propto = 1$. In the following sections, we compare models with specific choices of K and U based upon the type of functional forms that were derived in [1,2] – although not exactly the same, at this exploratory stage we simplify these functions a little.

NOMENCLATURE

p	=	Pressure field
x	=	Spatial variable
t	=	Time variable
K	=	Apparent diffusivity
U	=	Apparent velocity
R	=	External forcing
α	=	Fractional order

1 Nonlinear Transport Systems

1.1 Mathematical Problem

We consider solutions of the fractional system in equation (1) in an infinite domain, $(-\infty, \infty)$. In this work we consider only unforced systems, i.e. $R = 0$.

The boundary conditions are:

$$\lim_{x \to -\infty} p(x,t) = 1, \qquad \lim_{x \to \mp\infty} p(x,t) = 1,$$

And the initial condition is:

$$p(x,0) = 1 - e^{-x^2}.$$

We consider several different nonlinearities for the pair of apparent diffusivity K and apparent velocity U, in most cases they are strongly pressure dependent:

Case A: $K(p) = p,\ U(p, p_x) = p$

Case B: $K(p) = p,\ U(p, p_x) = pK$

Case C: $K(p) = p,\ U(p, p_x) = p\frac{\partial p}{\partial x}$

The advection term in each of these cases can be further simplified to:

In Case A, $\frac{\partial}{\partial x}(Up) = \frac{\partial}{\partial x}(p^2)$.
In Case B, $\frac{\partial}{\partial x}(Up) = \frac{\partial}{\partial x}(p^3)$.
In Case C, $\frac{\partial}{\partial x}(Up) = \frac{\partial}{\partial x}\left(p^2 \frac{\partial p}{\partial x}\right) = \frac{1}{3}\frac{\partial^2(p^3)}{\partial x^2}$

Case C involves a second order derivative, and we expect that this system may behave closer to a diffusive system.

In each of the 3 Cases, we simulate solutions for different fractional order, namely for,

$$\propto = 0.25, 0.5, 0.75, 1.0.$$

1.2 Numerical Method

The systems in equation (1) are nonlinear, and therefore needs special consideration in a numerical solver. A finite volume method was adopted, these methods readily integrate the divergence terms exactly across each control volume. The discretized system produces a tri-diagonal system of nonlinear algebraic equations, which is written,

$$\boldsymbol{A}(\boldsymbol{p})\boldsymbol{p} = \boldsymbol{S}(\boldsymbol{p}) \qquad (2)$$

where $\boldsymbol{A}$ is the coefficient matrix, $\boldsymbol{S}$ is the source vector, and $\boldsymbol{p}$ is the pressure vector for which we are solving. Equation (2) is nonlinear and must be linearized at each time step, t_n, such that $\boldsymbol{A}^{\boldsymbol{v}} = \boldsymbol{A}(\boldsymbol{p}^{\boldsymbol{v}}(x, t_{n-1}))$ and $\boldsymbol{S}^{\boldsymbol{v}} = \boldsymbol{S}(\boldsymbol{p}^{\boldsymbol{v}}(x, t_{n-1}))$, where v is the iteration counter,

$$\boldsymbol{A}^{\boldsymbol{v}}\boldsymbol{p}^{\boldsymbol{v}+\boldsymbol{1}} = \boldsymbol{S}^{\boldsymbol{v}} \qquad (3)$$

This is solved iteratively for the pressure vector, to convergence, $\lim_{v \to \infty} p^{v} \to p(x, t_n)$, before proceeding to the next time step at t_{n+1}. See [1] for details.

2 RESULTS

2.1 Nonlinear diffusion, $K = p, U = 0$

We start with a simple nonlinear diffusion system. The diffusivity is $K = p$. Figures 1(a)-1(c) show the simulations at different times, $0 \le t \le 1$, for different fractional orders as shown. The case $\propto = 1$ corresponds to the conventional case.

For all cases, we observe the initial spike in pressure at $x = 0$, diffuse and smoothen out in time, as we would expect. For higher fractional orders, $\propto = 1, \text{and } 0.75$, there does not appear to be a great deal of difference in the profiles at different times; but there is a marked acceleration in the smoothening out process in the case with $\propto = 0.5$. Thus fractional systems viewed as a as a function of the fractional order do not necessarily display a continuous and linear transition – this echoes the findings by Malik et al. [12].

2.2 A: Nonlinear advection-diffusion, $K = p, U = p$

We consider Case A, $K(p) = p,\ U(p, p_x) = p$, yielding:

$$\frac{\partial^{\propto} p}{\partial t^{\propto}} = \frac{\partial}{\partial x}\left(-p^2 + p\frac{\partial p}{\partial x}\right), \qquad t > 0, \quad (4)$$

Figures 2(a)-2(d) show the simulations at different times, $0 \le t \le 1$, for different fractional orders as shown. The figures are shown as 3D plots of $p(x,t)$ against x and t. This is a genuine advective and diffusive system, and the effects of both advection and diffusion appear to increase

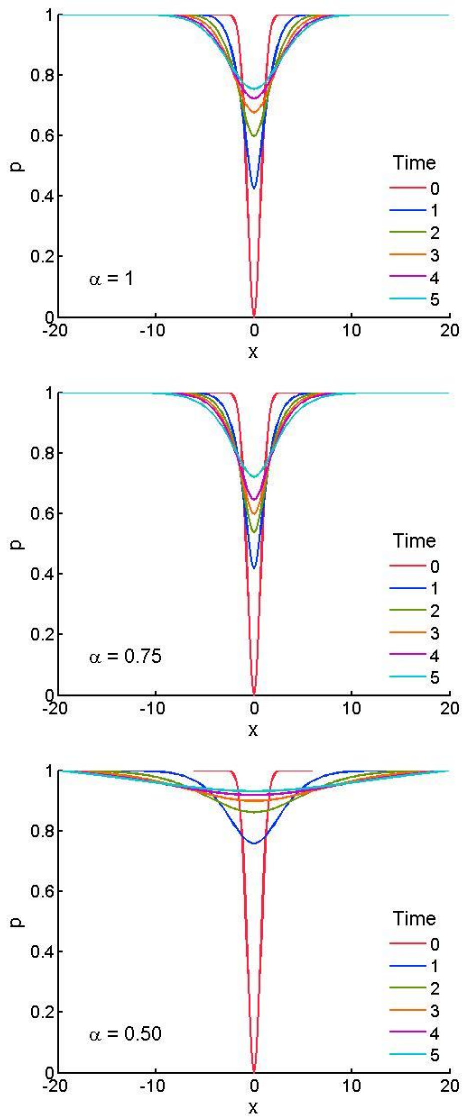

Figure 1: ***(a),*** $\propto = 1$; *(b)* $\propto = 0.75$; *(c)* $\propto = 0.5$

rapidly with decreasing fractional order $\propto$ -- in the limit of small $\propto$, the diffusion rapidly smoothens out the gradients in p, and at the same time, the spike in the pressure is transported more rapidly to the left.

2.3 Nonlinear advection-diffusion, $K = p, U = pK$

We consider Case B, $K(p) = p,\ U(p, p_x) = pK$, yielding:

$$\frac{\partial^{\propto} p}{\partial t^{\propto}} = \frac{\partial}{\partial x}\left(-p^3 + p\frac{\partial p}{\partial x}\right), \qquad t > 0, \qquad (5)$$

Qualitatively this appears to be a similar type power-law advective system to the previous Case A, 2.2. Figures 3(a)-3(d) show the simulations at different times, $0 \le t \le 1$, for different fractional orders as shown. The figures are shown as 3D plots of $p(x,t)$ against x and t. There are striking similarities to Case A: both the advection and the

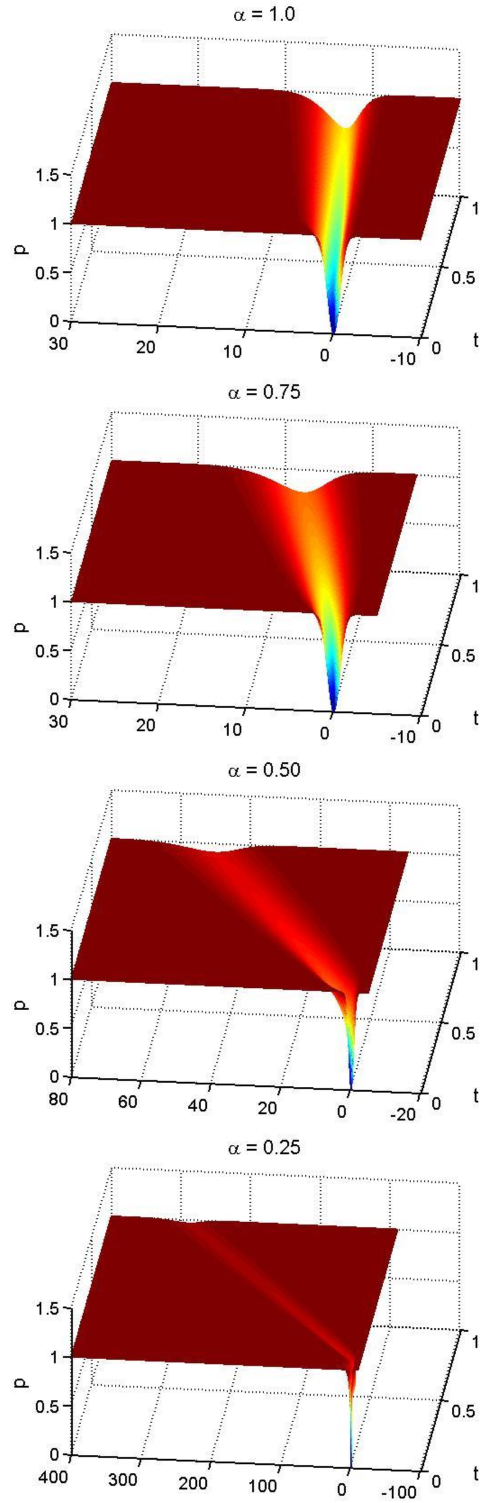

Figure 2: Case A. *(a),* $\propto = \mathbf{1}$***;*** ***(b)*** $\propto = \mathbf{0.75}$***;*** ***(c)*** $\propto = \mathbf{0.5}$, ***(d)*** $\propto = \mathbf{0.25}$

diffusion accelerate with decreasing fractional order $\propto$, and even quantitatively the plots looks similar.

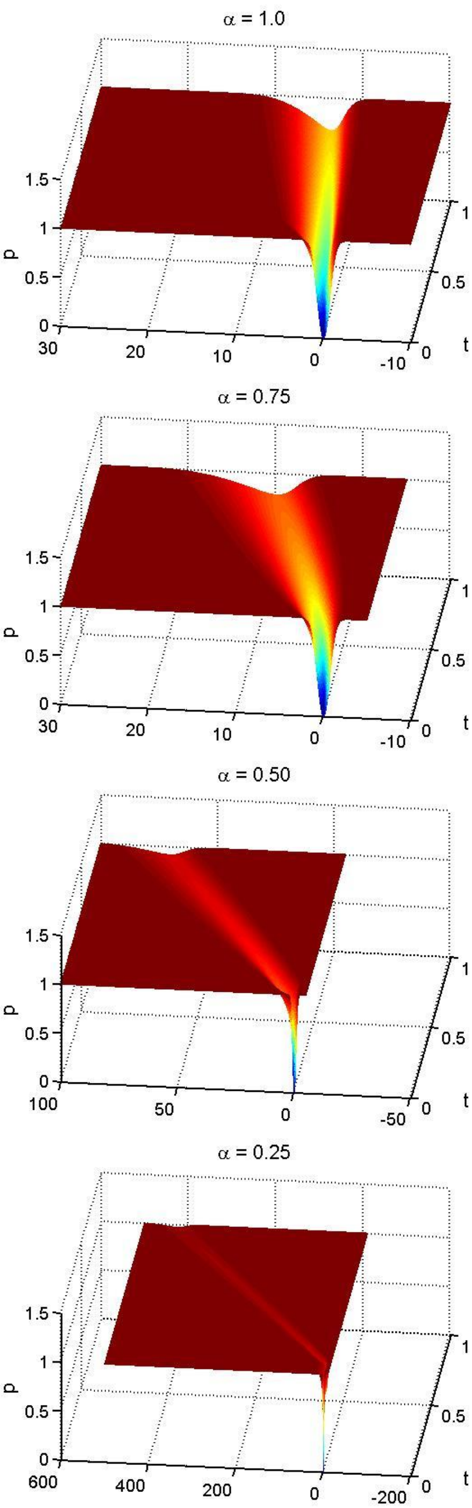

Figure 3: Case B. *(a)*, $\propto = \mathbf{1}$***; (b)*** $\propto = \mathbf{0.75}$***; (c)*** $\propto = \mathbf{0.5}$***, (d)*** $\propto = \mathbf{0.25}$

2.4 Nonlinear advection-diffusion, $K = p, U = p\frac{\partial p}{\partial x}$

We consider Case C, $K(p) = p,\ U(p, p_x) = pK$, yielding:

$$\frac{\partial^{\propto} p}{\partial t^{\propto}} = \frac{\partial^2}{\partial x^2}\left(-p^3/3 + p^2/2\right), \qquad t > 0, \qquad (6)$$

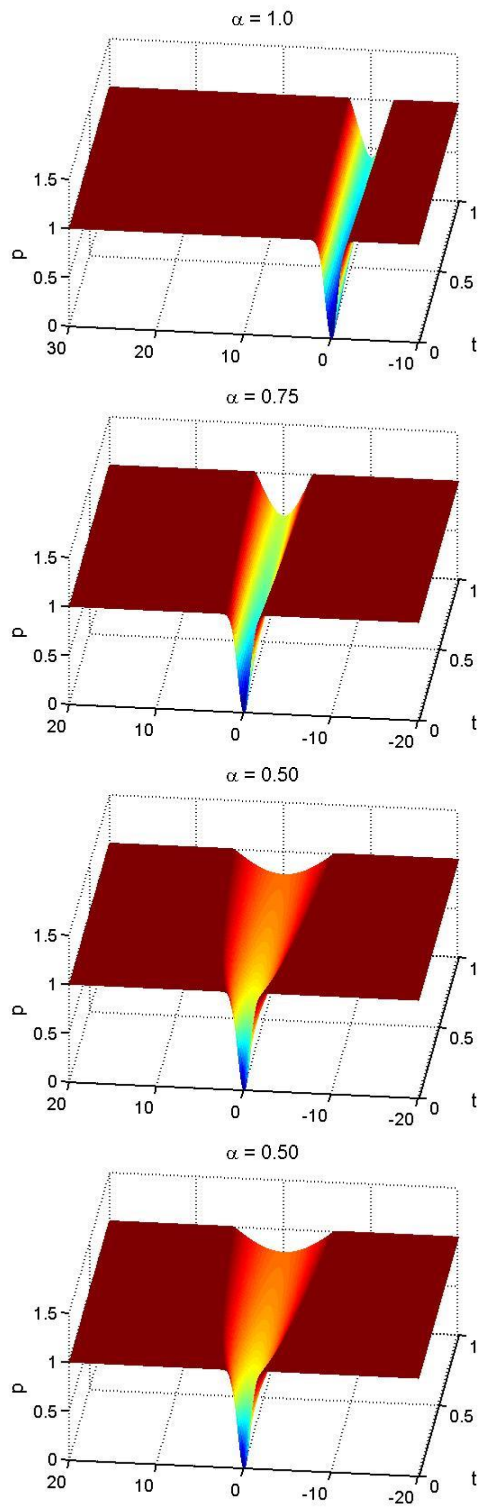

Figure 4: Case C. *(a)*, $\propto = \mathbf{1}$***; (b)*** $\propto = \mathbf{0.75}$***; (c)*** $\propto = \mathbf{0.5}$***, (d)*** $\propto = \mathbf{0.25}$

This appears to be a diffusion-like system. Figures 4(a)-4(d) show the simulations at different times, $0 \leq t \leq 1$, for different fractional orders as shown. The figures are shown as 3D plots of $p(x,t)$ against x and t, and indeed we se that the advection has disappeared and see what looks qualitatively like a purely diffusive process, similar to Figure 1. The rate of spread increases with decreasing fractional order.

2.5 Summary of different cases

Figure 5 summaries the main results from fractional systems in Cases A, B, and C in this work. The Cases A and B are very similar because the advection term is not proportional to the pressure gradient. Case C is close a genuine diffusion system because the advection term is proportional to the pressure gradient.

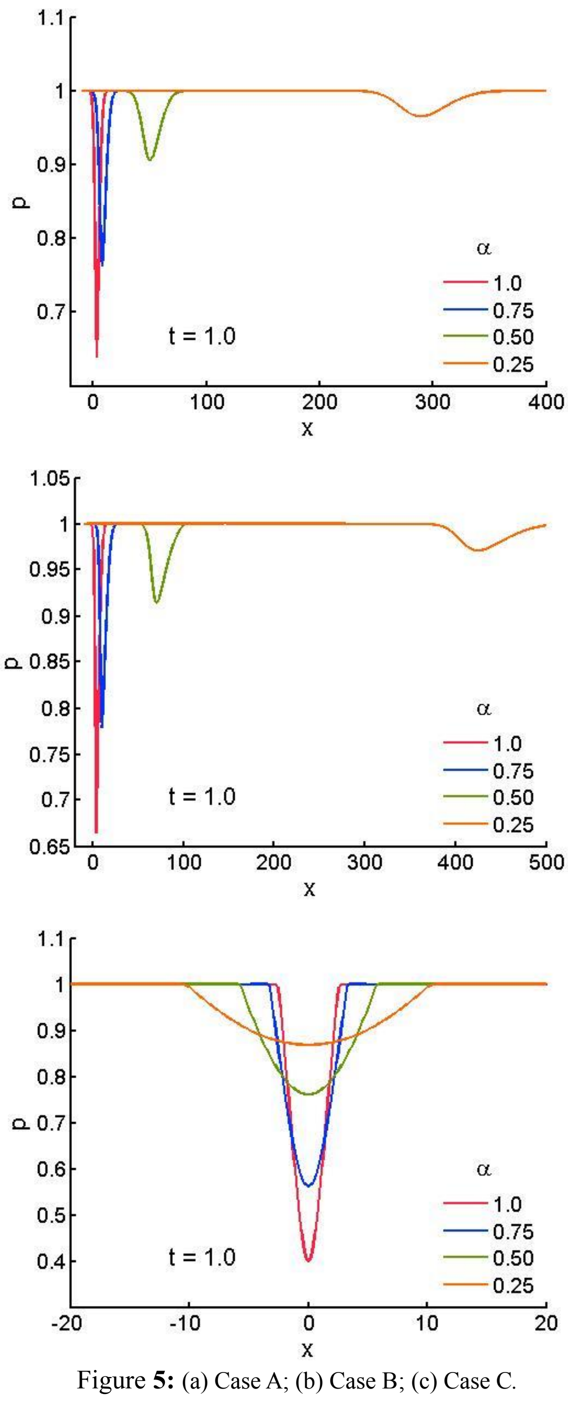

Figure **5:** (a) Case A; (b) Case B; (c) Case C.

CONCLUSIONS

Nonlinear fractional advection-diffusion systems have been investigated. Such systems form a simplified models for transport of gas through tight porous media such as shale gas in unconventional reservoirs. Fractional methods may be more effective at representing the flow in small but finite control volumes inside rocks, somewhat like a sub-grid scale model. The pressure distributions from such models over a period of time depends strongly upon the nonlinear models for the apparent diffusivity K and apparent velocity U, while the quantitative strength of the advective and diffusive effects depends upon the fractional order.

Critical issues to addressed in future is, firstly to identify the correct physical models for K and U, and secondly how to calculate or measure the correct fractional order for a given physical system and given scale.

ACKNOWLEDGEMENT

The authors would like to acknowledge the support provided by King Abdulaziz City for Science and Technology (KACST) through the Science Technology Unit at King Fahd University of Petroleum & Minerals (KFUPM) for funding this work through project No. 14-OIL280-04.

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