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QUANTIFICATION OF EFFECT OF CONVERGENCE IN POROUS MEDIA FLOW

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ABSTRACT

An attempt is made in this study to quantify the effect of convergence on macroscopic scale in the case of flow through porous media. Experiments are conducted separately on specially conceived parallel flow permeameter and converging flow permeameter keeping identical inlet and outlet conditions, using eight sizes of coarse granular media and water as the fluid medium. The media is sieved through sieves of different sizes to separate the crushed rock into sizes of 3.25 mm, 4.73 mm, 10.00 mm, 11.64 mm, 13.10 mm, 20.10 mm, 28.90 mm and 39.50 mm and glass spheres of 15.41 mm, 18.03 mm and 28.37 mm. As the effect of convergence is predominant in non- Darcian zones of flow, such as flow near the well, flow through rock fills, filters etc.,, the scope of the present work is restricted to flow regime with $R_e > 10$. (After Kovacs) Forchheimer's equation ($i = \mathbf{a}V + \mathbf{b}V^2$) is applied to analyze the experimental data. Equations are derived for Darcy parameter (**a**) and Non-Darcy parameter (**b**) of the Forchheimer's equation for the crushed rock and glass spheres by relating to size of the media (d) in both parallel flow condition and converging flow condition. From the results it is inferred that for a given rate of flow through a known size of aquifer having predetermined grain size, the resistance to flow is higher in the parallel flow compared to similar media conditions in converging flow configurations. A comparison is then made between the coefficients of the equation, computed for parallel and converging configurations of flow. The difference in these values is expressed in terms of a factor called **'Integrated Convergence factor (C_{fi})'**. It is concluded that the convergence of stream lines of seepage flow has a clear and profound influence on the relationship between resistance and regime. In order to make the findings reliable and suitable to field applications, the derived expressions are subjected to corrections for porosity effect, wall effect and tortuosity effect. Expressions for integrated convergence factor for crushed rocks and glass spheres are $C_{fi} = 1.095\ d^{-0.079}$ and $C_{fi} = 0.802\ d^{-0.25}$ respectively.

INTRODUCTION

In view of the significant contribution made by groundwater resources to water supply, any fact contributing to a greater understanding of the problems relating to groundwater flow is of prime concern. Ever since Darcy described his experiments in 1856, the occult subject of seepage flow has been subjected to continuous exploitation in both theoretical and experimental aspects. A steadily increasing interest has been created during the past century to study laws governing the flow of fluids through beds of granular media. Coupled with ever increasing demand for information brought about by the advances in technical sciences, many theoretical and experimental investigations have been carried out to establish the true relationship among different variables.

The concept of seepage flow and the final results are needed not in one discipline but in many diversified fields. Extraction of water from artesian basins by deep wells is a problem of in the flow of liquids through porous rocks or sands. In ground water hydrology it is needed to design the water supply, irrigation and drainage systems; in petroleum engineering gas and oils are to be developed from the underground reservoirs. Behavior of seepage flow is equally important in some specific applications in Civil Engineering, such as design of filter beds; flow through, around and beneath hydraulic structures. Study of diffusion and flow of fluids through materials such as bricks and porous earthen ware has been a problem in the ceramic industry. Scientific treatment of problems of irrigation, soil erosion and tile drainage is still open to further development.

It is common practice to solve these problems using Darcy's law, which is expressed as

$$V = ki \tag{1}$$

where V = superficial velocity of flow, k = permeability , i = hydraulic gradient.

It is generally accepted that Eq.(1) is valid for low Reynolds Numbers and at higher values of Re the linear relationship between V and i no longer holds good and exhibit non-linear relationship. Further, the non-linear nature of variation between hydraulic gradient and velocity of flow becomes more pronounced as the velocity increases rapidly when the boundaries are of converging configuration. Some field situations wherein the use of such non-linear relationship becomes necessary, in converging boundaries, are:

i. flow through filters used in water purification plants,
ii. flow through rock fill banks and dams with inbuilt spillways,
iii. flow in the area adjacent to pumping well, especially in a coarse grained aquifer,
iv. flow in the filter packs of tube wells.

For the sake of simplicity and to avoid cumbersome expressions, in general, the streamlines representing the direction of flow are assumed to be parallel, though such are less in common. They either will be converging or diverging.

An attempt is made in this paper to study the effect of convergence on the flow pattern and relating it to flow behavior. Quantification of this factor in terms of measurable parameters is included.

1 Forchheimer Equation

Forchheimer from his experiments on sand model for well flow was the first to propose an equation covering linear and non-linear ranges in a quadratic form as

$$i = aV + bV^2 \qquad (2)$$

in which a and b are coefficients determined by the properties of the fluid and porous medium and are known as Darcy and non- Darcy parameters. It is obvious form the above equation that 'aV' represents the rate of energy loss in the linear regime and 'bV^2' is that obtained in fully developed turbulent regime. Equation (2) was later refined by adding a third term as

$$i = aV + bV^2 + c\,V^3 \qquad (3)$$

A form proposed was

$$i = aV + bV^2 + c\,V^{1.5} \qquad (4)$$

The third term in Eqs.(3) and (4) accounts for transitional conditions of flow.

Equation (2) was further generalized to contain a time dependent term after Polubarinova – Kochina as

$$i = aV + bV^2 + c\,\frac{dv}{dt} \qquad (5)$$

For steady flow conditions, Eq.(5) reduces to Eq.(2). Though Eqs. (3) and (4) seem to be the more representative ones containing linear, turbulent and transitional regimes, according to Mc Corquodale (1969), this equation was slightly better than the two term equation. According to him, in the range of Reynolds number 600-4000, Eq. (3) was found to yield almost the same values of i as those computed using Eq. (2). A lot discussion can be found on different forms of Forchheimer Equation in the available literature. In general, the form represented by Eq.(2) is widely in computations because of its simplicity and reliable accuracy from the field point of view. In the present study also Eq.(2) is used to analyze the experimental data obtained from parallel and converging flow permeameters.

2 Experimentation

2.1 Permeameters

In order to achieve the objective of quantification of effect of convergence, two specially conceived permeameters viz., parallel flow and converging flow permeameters, were fabricated, the details of which are presented in Figs. 1 and 2.

Parallel flow permeameter (Fig.1), a G.I. column with 150 mm internal diameter , has a constant section through out the length of 6000 mm. The test section is confined to central 5000 mm with allowances of 500 mm each at the entrance and at the exit of the section. This is done to avoid possible effects of turbulence, due to the presence of porous screens kept at the entrance and exit. It may be found, in the past studies that only one test length is taken for computing hydraulic gradient. In the present study, three sets of test lengths are considered for computing average hydraulic gradient, which is expected to take into account the possible non-uniform packing of media in the permeameter. A row of piezometers provided on the surface of permeameter enabled noting down the head loss readings.

Converging flow permeameter (Fig.2) with a central angle of 0.70 rad (40.70^0) and 500 mm wide (perpendicular to plane of paper) has front and rear faces made of 12.50 mm thick Perspex sheet. The tapering sides of permeameter are made of 6.0 mm M.S. Sheet, to which bearings were fitted to facilitate overturning and thus to fill or remove the porous media. Two curved perforated screens with more than 85% perforations were placed one each at the entrance and exit of the section facilitated radial flow and uniform spreading of the flow through the media. A row of piezometers was provided along the front face of the permeameter to note down the head loss readings.

In both the cases, header tanks ensured turbulent-free entry of water into the test section. Horizontal perforated pipes fitted at the exit end of delivery pipe, allowed water to fall in the form of shower instead a single thick jet, to ascertain the uniform entry conditions at the entry of the permeameter form the header tank. Two valves of 75 mm dia are fitted at the supply point and discharge end of the permeameter to regulate the flow through the permeameter. Further, at the discharge end, a bypass valve is provided to facilitate fine regulation of low and to maintain steady conditions. Using a pre calibrated triangular notch discharges were recorded to an accuracy of ±0.25% at higher heads and to an accuracy of ± 2.50% at lower heads.

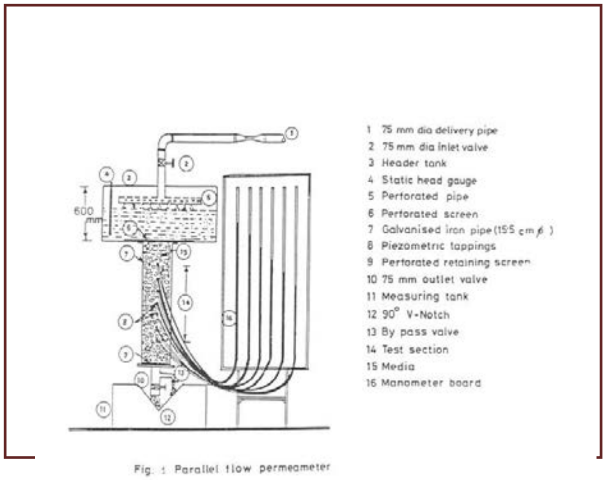

Figure 1 Parallel Flow Permeameter

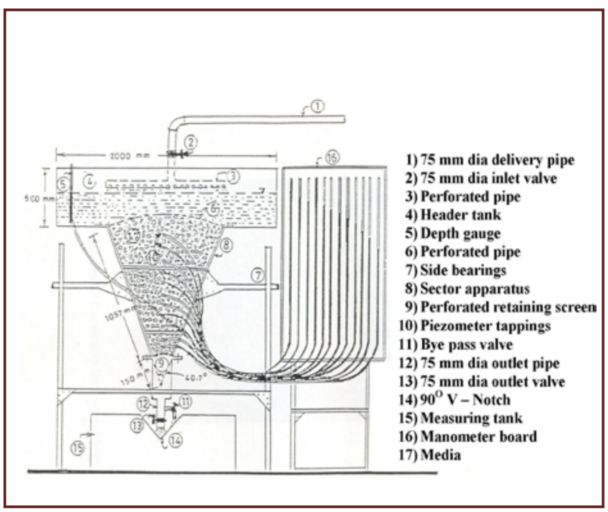

Figure 2 Converging Flow Permeameter

3 Determination Of Media Parameters

3.1 Size

Crushed rock of 3.25 mm, 4.73 mm, 10.00 mm, 11.64 mm, 13.10 mm, 20.10 mm, 28.90 mm and 39.50 mm and glass spheres of 15.41 mm, 18.03 mm and 28.37 mm size are used as media. In the present analysis, 'volume diameter', that is, diameter of a sphere having same volume as that of the irregular shaped particle, is used to denote the size of the medium and it is determined by water displacement method.

3.2 Porosity

It is a very sensitive parameter in porous media flow. It is determined as follows: Permeameter is cleaned, dried and then filled with the medium of known size up to the top. Outlet valves are closed and the permeameter is slowly filled until water level reaches the lowest piezometer. A measured quantity of water is then poured till water reaches top piezometer. This measured volume of water indicates the volume of voids between top and bottom piezometers. From geometry, volume of the permeameter enclosed between these two piezometers is computed from which porosity is computed.

4 Experimental Procedure

Permeameter is filled with the medium, under gravity, ensuring even packing by varying height of fall uniformly. Water is then allowed to flow through the permeameter under a constant head for a period of 1 to 1.5 hours at maximum possible rate so that all the particles are reoriented and no further reorientation takes place during experimentation. Before taking the piezometer readings, it is ensured that all the entrapped air is removed. Once the flow attains steady state conditions, discharge and the corresponding head loss readings are noted. As the water level in the piezometer fluctuates, three pairs of maximum and minimum readings are taken and difference in average of these readings is taken as the head loss. During every run, temperature of the outflow is noted, from which viscosity is determined.

4.1 Velocity of Flow

Velocity of flow is the basic dynamic dependent variable which controls the entire analysis. In the case of converging flow, area of cross section varies along the length of travel and hence, velocity of flow becomes space dependent. At any radius r_c from the centre of convergence, the superficial velocity of flow (V) is given by,

$$V = \frac{Q}{A_c} \tag{6}$$

in which $A_c = r_c \cdot \theta \cdot w$ (7)

where Q = discharge , A_c = Area of flow at a section of radius of convergence, r_c, from the centre of convergence, θ = Central angle of convergence in radians ; w = Width of flow between two parallel confining surfaces.

4.2 Hydraulic Gradient

In the case of converging flow, as velocity varies from point to point, hydraulic gradient also is a spatial function. Therefore, a separate procedure is needed to compute the hydraulic gradient at a point, unlike parallel flow conditions, wherein its value is assumed to be constant.

Head loss (dh_L) over a length (dx) may be written as,

$$dh_L = \frac{f_1 dx V^n}{2 g d_p} \tag{8}$$

where d_p is the pore diameter, g is gravitational constant and f_1 is friction coefficient.

The left hand side of the above equation represents the hydraulic gradient as a function of distance of travel x. Therefore,

$$i_x = K_1 e^{mx} \tag{9}$$

The values of K_1 and m are obtained from experimental data by the method of least squares for a given size of the medium and for a known rate of flow.

5 Analysis Of Experimental Data And Results

5.1 Analysis Of The Data Obtained From Parallel Flow Permeameter:

Various steps followed in this study are in the following order:

(i) Grouping of experimental data into Darcy and non-Darcy regimes.
(ii) Verification and ascertaining reliability of trend of present experimentation with that of past.
(iii) Examining the applicability of Forchheimer equation to the data and to study variation of the coefficients **a** and **b** with size of medium.
(iv) Estimation of porosity, wall and tortuosity corrections and incorporating the modifications necessary in the equations developed therein.

Of all the forms, most widely used form is that proposed by Forchheimer , which is

$$i = \mathbf{a}V + \mathbf{b}V^2 \tag{10}$$

or

$$i/V = \mathbf{a} + \mathbf{b}V \tag{11}$$

which is similar in form to

$$Y = c_o + m_o x \tag{12}$$

which is an equation of a straight line.

Comparing corresponding terms of Eqs. (11) and (12), when a plot is made between i/V on y-axis and V on x-axis, then the data must lie along a straight line, with linear parameter **a** equal to y-intercept (i/V intercept) and non-linear parameter **b** is equal to slope of (i/V vs V) line.

Data obtained from present study and the studies conducted by Nasser (1970) and Niranjan (1973) have been combined and analyzed to obtain a relationship between size of the medium and the coefficients **a** and **b** for coarse media. Application of Forchheimer equation for the data with $R_e \geq 10$ is examined. Corrections for porosity, wall and tortuosity effects are applied to experimental data. Once again, for these corrected experimental data, Forchheimer equation is applied. Expressions relating Darcy and non Darcy parameter with size of the medium have been obtained.

Parameters $\mathbf{a_{pc}}$ and $\mathbf{b_{pc}}$ for coarse granular media are related to size **d** by the equations :

$$(\mathbf{a_{pc}})_{cr} = \frac{0.0053}{d^{1.288}} \tag{13}$$

$$(\mathbf{b_{pc}})_{cr} = \frac{0.0017}{d^{1.095}} \tag{14}$$

Equations for $\mathbf{a_{pc}}$ and $\mathbf{b_{pc}}$ in terms of size for glass spheres are :

$$(\mathbf{a_{pc}})_{gs} = \frac{0.0033}{d^{1.207}} \tag{15}$$

$$(\mathbf{b_{pc}})_{gs} = \frac{0.0002}{d^{0.3697}} \tag{16}$$

The suffix 'pc' denotes the values of different parameters in the parallel flow permeameter after applying corrections for porosity, wall and tortuosity effects.

5.2 Analysis Of The Data Obtained From Converging Flow Permeameter :

Various steps followed in this study are in the following order:

(i) Bringing out the nature of converging flow
(ii) Examining the applicability of Forchheimer equation to the data and to study variation of the coefficients $\mathbf{a_c}$ and $\mathbf{b_c}$ with size of medium.
(iii) Estimation of porosity, wall and tortuosity corrections and incorporating the modifications necessary in the equations developed therein.

Parameters $\mathbf{a_{cc}}$ and $\mathbf{b_{cc}}$ for coarse granular media are related to size d by the equations :

$$(\mathbf{a_{cc}})_{cr} = \frac{0.0054}{d^{0.943}} \tag{17}$$

$$(\mathbf{b_{cc}})_{cr} = \frac{0.0003}{d^{1.045}} \tag{18}$$

Equations for $\mathbf{a_{cc}}$ and $\mathbf{b_{cc}}$ in terms of size for glass spheres are :

$$(\mathbf{a_{cc}})_{gs} = \frac{0.0010}{d^{1.039}} \tag{19}$$

$$(\mathbf{b_{cc}})_{gs} = \frac{0.0003}{d^{0.7426}} \tag{20}$$

The suffix 'cc' denotes the values of different parameters in the converging flow permeameter after applying corrections for porosity, wall and tortuosity effects.

6 Quantification Of Effect Of Convergence – Convergence factor :

As, the present study is confined to only flow through converging boundaries, an attempt is made in this section to quantify the effect of convergence.

The equations pertaining to parallel flow conditions are fairly different from those of converging flow configuration. As all the equations have been standardized for porosity, wall and tortuosity effect, the difference may be attributed to the effect of convergence in the range of experiments conducted ($0.12 < R_e < 14725$).

Treating parallel flow conditions as reference, resistance offered to flow of a fluid through a porous medium confined in converging flow configuration may be expressed in terms of parallel flow conditions. That is, for coarse granular media, ($\mathbf{a_{cc}}$) can be expressed in terms of ($\mathbf{a_{pc}}$). Similarly, ($\mathbf{b_{cc}}$) can be expressed in terms of ($\mathbf{b_{pc}}$). In the case of glass spheres also, parameters pertaining to converging flow can be related to corresponding parameters of parallel flow conditions. A glance at the equations cited in Sec. 5.1and 5.2, leads to the fact that these parameters in turn are expressed in terms of size of the media. Hence, a comparison of these parameters for both the configurations in terms of size will make analysis lucid and practically useful on the field. From the plots it is obvious that convergence of flow has a definite bearing on reducing the resistance to flow.

In order to quantify the effect of convergence on the resistance to flow, a dimensionless parameter called '***Integrated Convergence Factor***' ($\mathbf{C_{fi}}$) is defined as follows :

$$C_{fi} = C_{fa} + C_{fb} \tag{21}$$

where C_{fi} = Integrated convergence factor
C_{fa} = Darcian convergence factor
C_{fb} = non-Darcian convergence factor

Effect of convergence in the Darcian term of Forchheimer equation is expressed as

$$C_{fa} = \frac{a_{pc} - a_{cc}}{a_{pc}} \tag{22}$$

$$= 1 - \frac{a_{cc}}{a_{pc}} \tag{23}$$

$$\mathbf{C_{fa} = 1 - 0.625\, d^{0.098}} \tag{24}$$

On similar lines for non Darcy parameter

$$C_{fb} = \frac{b_{pc} - b_{cc}}{b_{pc}} \tag{25}$$

$$= 1 - \frac{b_{cc}}{b_{pc}} \tag{26}$$

$$\mathbf{C_{fb} = 1 - 0.0645\, d^{0.0211}} \tag{27}$$

Therefore, Eq.(21) becomes

$$C_{fi} = 1.095\ d^{-0.079} \qquad (28)$$

An expression for integrated convergence factor (C_{fi}) for glass spheres is obtained as

$$C_{fi} = 0.802\ d^{-0.25} \qquad (29)$$

Therefore, using Eq.(28) and (29) the effect of convergence can be computed for a given size of coarse granular and spherical shaped media respectively.

CONCLUSIONS

From the results it is inferred that for a given rate of flow through a known size of aquifer having predetermined grain size, the resistance to flow is higher in the parallel flow compared to similar media conditions in converging flow configurations. It can also be concluded that for a known size of the medium packed in converging flow configuration, either for coarse media or round particles and for a given rate of flow, corresponding head loss can be determined , as 'a_{cc}' and 'b_{cc}' provide measures of energy loss. Forchhiemer's coefficients, which are representatives of hydraulic gradient, for both the configurations are compared and it is concluded that effect of change in configurations is very clear and influences the magnitude of resistance to flow through a known size of the media. The effect of convergence is quantified in terms of a new factor '***Convergence factor***' ***(C_f)***. An attempt is made to express the convergence factor in terms of measurable parameters.

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