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# NUMERICAL SOLUTION OF BURGERS' EQUATION ARISING IN LONGITUDINAL DISPERSION PHENOMENON IN FLUID FLOW THROUGH POROUS MEDIA BY CRANK-NICOLSON SCHEME

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## ABSTRACT

The present paper discusses the numerical solution of the Burgers' equation arising in longitudinal dispersion phenomenon in fluid flow through porous media. In the porous medium pure water, salt water or contaminated water disperse in longitudinal direction gives rise to a non-linear partial differential equation in the form of Burgers' equation. The equation is solved by using Crank-Nicolson finite difference scheme with appropriate initial and boundary conditions. The longitudinal dispersion phenomenon may be miscible or immiscible fluid flow through porous media. The problem of miscible displacement can be seen in the coastal areas, where fresh water beds are gradually displaced by sea water. Longitudinal dispersion phenomenon plays an important role to control salinity of the soil in western seashore region of the Gujarat state in India. To control salinity, the government of Gujarat has developed many check dams from where contaminated water diverted and poured to the soil of the farms, where the crops of cumin seed (jeera), fennel (saunf) and other grains are grown. In this region due to the infiltration of this infiltrated water, free surface of sweet water table rises, consequently, saline seawater cannot cross the threshold in the nearby area of the seashore. In such a way, the dispersion of contaminated water plays key role to solve salinity problem. The immiscible dispersion also plays an important role in petroleum engineering during secondary oil recovery process, in which water or gas is injected in oil formatted area to drive the oil towards production well. An unconditionally stable Crank-Nicolson finite difference scheme has been employed to find the concentration  $C(X, T)$  of salty or contaminated water dispersion in uni-direction. The outcome is consistent with physical phenomenon of longitudinal dispersion in miscible fluid flow through porous media. It is concluded, that the concentration  $C(X, T)$  decreases as distance  $X$  as well as time  $T$  increases. The tables and graphs are developed by using MATLAB coding.

## INTRODUCTION

The region of the soil that is unsaturated is known as the vadose zone (or unsaturated zone), and this is the region where the most interesting nonlinear hysteretic behaviour is observed. As shown in the Fig. 1 saturated zone is one in which the entire void space is occupied by water. In the

unsaturated zone, only part of the void space is occupied by water and the remainder being occupied by a gaseous and possibly by a non-aqueous phase liquid. In saturated ground water aquifers, all available pore spaces are filled with water. Above a capillary fringe some part of pore spaces also contains air.

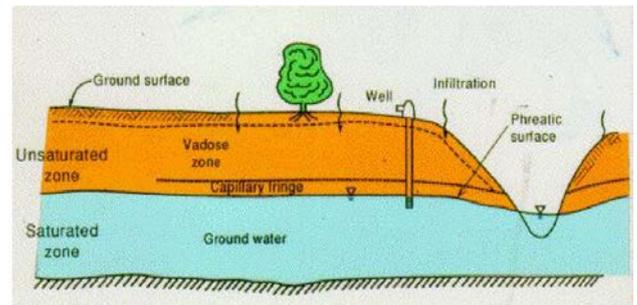


Figure 1: Natural and earth section view of saturated zone and unsaturated zone.

The mixing of viscous fluids is a most common incidence in chemical engineering applications. Similar processes encountered in environmental problems such as the spreading of aqueous or non aqueous pollutants following an accidental discharge. The mixing of these pollutants with surrounding water flows generally includes underground porous layers. And the dispersion is a macroscopic phenomenon caused by a combination of molecular diffusion and hydrodynamic mixing occurring with laminar flow through porous medium [2, 8, 17].

Analyzing the generic case of two fluids in contact flowing (simultaneous flow) through a porous medium it is observed that mixing is almost associated to the random walk of fluid (or tracer) particles through the disordered structure of the pore volume and thermal molecular agitation is dominant only at very low mean flow velocities. The steps of the random walk are much larger than those of thermal Brownian motion so that the corresponding spreading scale and the width of the dispersion front are correspondingly increased. Of course, the minimum size of heterogeneities of the mixture obtained in this way is also larger; however, if the medium is adequately homogeneous, this size is of the order of the grain diameter so that molecular diffusion can generally complete the mixing. This mixing is known as

hydrodynamic dispersion and it is referred to as mixing of miscible fluids.

The flow of groundwater in coastal aquifers, as shown in the Fig. 2, can be treated as an interface problem in which two fluids of different densities, fresh water and salt water, have a clear interface rather than a transition zone. This flow problem assumes that the fresh water flows over the salt water which is at rest [3, 17].

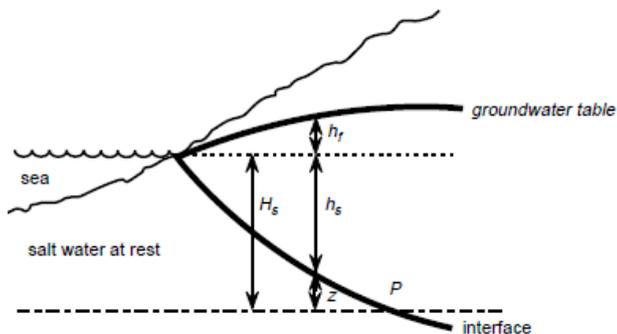


Figure 2: Interface flow in coastal aquifer

Saltwater intrusion, in which saline water displaces or mixes with fresh groundwater, is one of the major causes of groundwater contamination. Saltwater intrusion is usually caused when the hydrodynamic balance between the fresh water and the saline water is disturbed, such as when fresh groundwater is over pumped in coastal aquifers. Saltwater intrusion can also occur when the natural barriers that separate fresh and saline water are destroyed, such as in construction of coastal drainage canals that enable tidal water to advance inland and percolate into a freshwater aquifer.

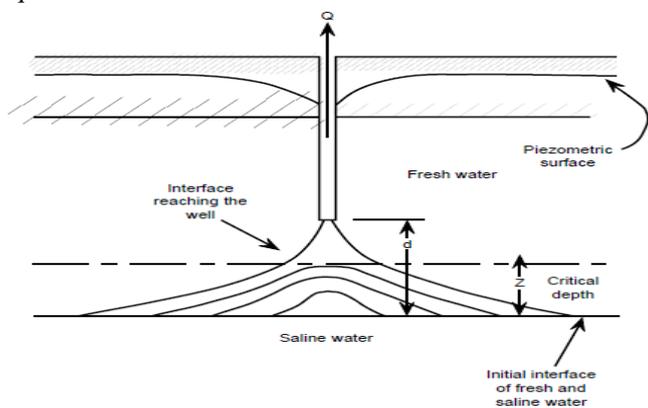


Figure 3: Upcoming of salt water under a pumping well.

Fig. 3 depicts a situation in which water is pumped from a freshwater zone under laid by a saline water layer. The interface between fresh water and saline water rises toward the well in a cone shape as shown in the Fig. 3.

Controlling the intrusion of saline water before it contaminates an aquifer system is desirable because removing it once it has developed is difficult. Years may require restoring normal conditions. In coastal aquifers, according to Ghyben-Herzberg, fresh water exists below sea level for every meter of fresh water above sea level [3, 8].

The problem of miscible displacement often observed in coastal areas, where the fresh water beds are gradually displaced by sea water. These day's efforts are being made by the environmentalist to dispose the atomic waste products born from nuclear reactor and dumped inside the ground by using the same phenomenon of displacement. Among Many flow problems in porous media, one involves fluid mixtures called miscible fluids. A miscible fluid is a single phase fluid consisting of several completely dissolved homogenous fluid species, a distinct fluid-fluid interface doesn't exist in a miscible fluid. The flow of miscible fluid is an important incidence in petroleum industry; an enhanced oil recovery technique in oil reservoir involves injecting a fluid (solvent) that dissolves into the reservoir's oil [1, 6, 9].

In a miscible displacement process a fluid is displaced in a porous medium by another fluid which is miscible with the first fluid. Miscible displacement in porous media plays an important role in many engineering and science fields such as oil recovery in petroleum engineering, contamination of ground water by waste product disposed, underground movement of mineral in the soil and recovery of spent liquors in pulping process[7, 10].

The key benefit of this research is to improve conceptual models how all contaminants migrate through homogeneous, variably-saturated, porous media. These problems of dispersion have been receiving considerable attention from chemical, environmental and petroleum engineers, hydrologists, hydro-geologist, mathematicians and soil scientists.

Most of the works reveal common assumption of homogenous porous media with constant porosity, steady seepage flow velocity and constant dispersion coefficient. For such assumption Ebach and White (1958) studied the longitudinal dispersion problem for an input concentration that varies periodically with time and Ogata and Banks (1961) for a constant input concentration. Hoopes and Herteman (1965) studied the problem of dispersion in radial flow from fully penetrating; homogenous, isotropic non-adsorbing confined aquifers. Bruce and street (1966) considered both longitudinal and lateral dispersion with in semi finite non adsorbing porous media in a steady unidirectional flow fluid for a constant input concentration. Marino (1978) considered the input concentration varying exponentially with time. Al-Niami and Rushton (1977) and Marino (1974) studied the analysis of flow against dispersion in porous media. Basak (1978) presents an analytical solution to the problem of Evaporation from a horizontal soil column in which diffusivity increases linearly with moisture content and also to a problem of concentration dependent diffusion with decreasing concentration at the source. Hunt (1978) applied the perturbation method to longitudinal and lateral dispersion in no uniform seepage

flow through heterogeneous aquifers. Wang (1978) discussed the concentration distribution of a pollutant arising from a instantaneous point source in a two dimensional water channel with non uniform velocity distribution. He employed Gill's method to solve the convective diffusion equation. Kumar (1983) discussed the Dispersion of Pollutants in Semi-Infinite Porous Media with Unsteady Velocity Distribution. Mehta and Patel (2005) applied Hope-Cole transformation to unsteady flow against dispersion of miscible fluid flow through porous media. Mehta and Saroj (2009) considered that the longitudinal dispersion coefficient is directly proportional to the concentration and the velocity component in direction of X-axis is considered directly proportional to ratio of distance  $x$  and time  $t$ . Also, this phenomenon has been discussed by Mehta & Meher (2010) using Backlund Transformations, Mehta & Joshi (2012) using Group theoretic approach. The present paper discusses the numerical solution of nonlinear partial differential equation for longitudinal dispersion phenomenon in  $x$ -direction which takes place when miscible fluids mix and flow in uni-direction. The mathematical formulation of the problem leads to a non-linear Burgers' equation whose numerical solution has been obtained by unconditionally stable Crank-Nicolson finite difference scheme.

## NOMENCLATURE

$C_0$	Initial input concentration of contaminant
$C$	Concentration of contaminant in liquid phase
$\rho$	Density of the mixture
$\bar{V}$	Pore seepage velocity vector.
$D_L$	Longitudinal dispersion coefficient $\gamma$ based on $u$
$t$	Time (s)
$x$	Linear distance coordinate (m)

## 1 Statement of the problem

Miscible displacement in porous media is a type of double-phase flow in which two phases are completely soluble in each other. Therefore, capillary forces between the fluids do not come into effect. The longitudinal dispersion of the contaminated or saline water with the concentration  $C(x, t)$  flowing in the  $x$ -direction has been considered, the homogeneous porous medium is saturated with fresh water. The miscible flow (contaminated or saline and fresh water) under conditions of complete miscibility could be thought to behave, locally at least, as a single-phase fluid, which would obey Darcy's law. The change of concentration, in turn, would be caused by diffusion along the flow channels and thus be governed by the bulk coefficients of diffusion of the one fluid in the other. There is no mass transfer between the solid and liquid phases, is assumed [2, 4, 5]. The miscible flow takes place both longitudinally and transversely, but

the spreading caused by dispersion is greater in the direction of flow than the transverse direction.

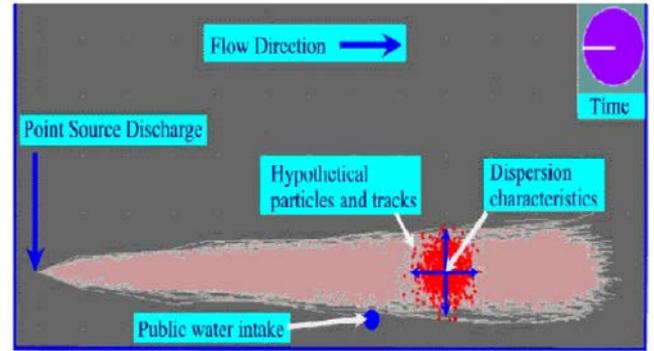


Figure 4: Longitudinal and Transverse dispersion

The problem is to describe the growth of the mixed region, i.e. to find concentration  $C(X, T)$  of the contaminated water as a function of time  $t$  and position  $x$ , as two miscible fluids flow through homogeneous porous media. Outside of the mixed zone (on either side) the single-fluid equation describes the motion of fluid.

## 2 Mathematical formulation of the Problem

In the present problem it is considered that the dispersion zone is in one direction i.e.  $x$ -direction. One dimensional treatment of dispersion phenomena avoids the radial or transverse component of dispersion.

According to Darcy's law, the equation of continuity for the mixture, in the case of incompressible fluids is given by Bear [2, 3].

$$\frac{\partial \rho}{\partial t} + \text{div} \cdot (\rho \bar{V}) = 0 \quad (1)$$

where,  $\rho$  is the density for mixture and  $\bar{V}$  is the pore seepage velocity vector.

The equation of diffusion for a fluid flow through a homogeneous porous medium, without increasing or decreasing the dispersing material is given by

$$\frac{\partial C}{\partial t} + \text{div} \cdot (C \bar{V}) = \nabla \cdot \left[ \rho \bar{D} \text{div} \left( \frac{C}{\rho} \right) \right] \quad (2)$$

where,  $C$  is the concentration of the fluid A into the other host fluid B (i.e.  $C$  is the mass of fluid A per unit volume of the mixture) and  $\bar{D}$  is the tensor coefficient of dispersion having unit  $[length^2 \cdot time^{-1}]$  with nine components  $D_{ij}$

. In a laminar flow through a homogeneous porous medium at constant temperature  $\rho$  may be considered as constant. Then

$$\text{div} \cdot \bar{V} = 0 \quad (3)$$

Hence equation (2) can be written as

$$\frac{\partial C}{\partial t} + \bar{V} \cdot \nabla C = \text{div} [\bar{D} \cdot \text{div} C] \quad (4)$$

When the seepage velocity  $\bar{V}$  is along the x-axis, the non-zero components are

$$D_{11} = D_L = \frac{L}{C_0^2} \quad (\text{Coefficient of longitudinal dispersion})$$

and  $D_{22} = D$  (Coefficient of transverse dispersion) and other  $D_{ij}$ 's are zero. In this case the equation (4) becomes [3, 5, 9, 11, 12, 14],

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} = D_L \frac{\partial^2 C}{\partial x^2} \quad (5)$$

Where,  $u$  is the component of flow velocity  $\bar{V}$  along the x-axis having the dimension  $[length \cdot time^{-1}]$  it is time dependent along the x-axis in the non-negative direction and  $D_L = \gamma > 0$ , it is the cross sectional flow velocity of porous medium.

By Mehta [12, 13, 14] it has been observed that seepage flow velocity  $u$  is related with concentration of the dispersing material as

$$u = \frac{C(x, t)}{C_0} \quad \text{for } x > 0 \quad (6)$$

where, the concentration of the contaminated water at  $x = 0$  is very high and it is constant  $C_0 \cong 1$  (Mehta 2006) By using equation (6) in the equation (5), we get,

$$\frac{\partial C}{\partial t} + C \frac{\partial C}{\partial x} = \gamma \frac{\partial^2 C}{\partial x^2} ; \quad 0 \leq x \leq L \quad (7)$$

where,  $\gamma$  is the coefficient of longitudinal dispersion.

The equation (7) is a governing non linear Burgers' equation for longitudinal dispersion of miscible contaminated water flow through porous media. As given in the statement, the dispersion is uni-directional displacements flow through semi-finite homogeneous porous media, the seepage flow velocity of contaminated water is assumed unsteady. Here the initial concentration of dispersion is considered as an input highest constant concentration of contaminants at  $x = 0$  is  $C_0$ . The porous medium is considered as non-adsorbing. The governing partial differential equation (7) for longitudinal hydrodynamic dispersion with in a semi finite non adsorbing porous medium in a unidirectional flow field in which  $\gamma$  is the longitudinal dispersion coefficient,  $C$  is the average cross-sectional concentration,  $u$  is the unsteady seepage velocity,  $x$  is a coordinate parallel to flow and  $t$  is time.

### 3 Numerical Solution of the problem using Crank-Nicolson Scheme

To make the equation (7) dimensionless, we set the dimensionless variables as

$$X = \frac{x}{L} \quad \text{and} \quad T = \frac{t}{L} \quad \text{so that} \quad 0 \leq X \leq 1, \quad 0 \leq T \leq 1.$$

Hence, the equation (7) reduces to

$$\frac{\partial C}{\partial T} + C \frac{\partial C}{\partial X} = \gamma \frac{\partial^2 C}{\partial X^2} ; \quad 0 \leq X \leq 1 \quad (8)$$

The appropriate initial and boundary conditions are taken as

$$\begin{aligned} C(X, 0) &= (1 - X)^2, \quad 0 < X \leq 1 \\ C(0, T) &= C_0 = 1, \quad 0 < T \leq 1 \\ C(1, T) &= C_1 = 0.001, \quad 0 \leq T < 1 \end{aligned} \quad (9)$$

The non-linear partial differential equation (8) has been discretized by using Crank-Nicolson finite difference scheme as follows [4, 15, 16, 18]:

To handle the non-linear co-efficient  $C$  the value of  $C_{i,n+\frac{1}{2}}$  at the half time level is

$$C_{i,n+\frac{1}{2}} = C_{i,n} + \frac{\Delta T}{2} \left[ -C_{i,n} \cdot \frac{(C_{i+1,n} - C_{i-1,n})}{2(\Delta X)} + \gamma \left( \frac{C_{i+1,n} - 2C_{i,n} + C_{i-1,n}}{(\Delta X)^2} \right) \right] \quad (10)$$

Let the ratio  $r = \Delta T / (\Delta X)^2$  expression (10) becomes,

$$C_{i,n+\frac{1}{2}} = C_{i,n} + \frac{r}{4} \left[ -C_{i,n} \cdot (C_{i+1,n} - C_{i-1,n}) \Delta X + 2\gamma (C_{i+1,n} - 2C_{i,n} + C_{i-1,n}) \right] \quad (11)$$

The Crank-Nicolson finite difference scheme of the equation (8) is as under: For  $2 \leq i \leq R - 1$

$$\begin{aligned} & \left[ \gamma + \frac{C_{i,n+\frac{1}{2}}}{2} \cdot \Delta X \right] C_{i-1,n+1} - 2 \left[ \gamma + \frac{1}{r} \right] C_{i,n+1} + \\ & \left[ \gamma - \frac{C_{i,n+\frac{1}{2}}}{2} \cdot \Delta X \right] C_{i+1,n+1} \\ & = - \left[ \gamma + \frac{C_{i,n+\frac{1}{2}}}{2} \cdot \Delta X \right] C_{i-1,n} + 2 \left[ \gamma - \frac{1}{r} \right] C_{i,n} \\ & \quad - \left[ \gamma - \frac{C_{i,n+\frac{1}{2}}}{2} \cdot \Delta X \right] C_{i+1,n} \end{aligned} \quad (12)$$

For  $i = 1$

$$C_{1,n+\frac{1}{2}} = C_{1,n} + \frac{r}{4} \left[ -C_{1,n} \cdot (C_{2,n} + C_{1,n} - 2) \Delta X \right] + 2\gamma (C_{2,n} - 3C_{1,n} + 2) \quad (13)$$

$$\begin{aligned} & - \left[ 3\gamma + \frac{2}{r} + \frac{C_{1,n+\frac{1}{2}}}{2} \cdot \Delta X \right] C_{1,n+1} + \\ & \left[ \gamma - \frac{C_{1,n+\frac{1}{2}}}{2} \cdot \Delta X \right] C_{2,n+1} \\ & = \left[ 3\gamma - \frac{2}{r} + \frac{C_{1,n+\frac{1}{2}}}{2} \cdot \Delta X \right] C_{1,n} - \\ & \left[ \gamma - \frac{C_{1,n+\frac{1}{2}}}{2} \cdot \Delta X \right] C_{2,n} - 2 \left[ 2\gamma + C_{1,n+\frac{1}{2}} \cdot \Delta X \right] \end{aligned} \quad (14)$$

For  $i = R$  the scheme is

$$C_{R,n+\frac{1}{2}} = C_{R,n} + \frac{r}{4} \left[ C_{R,n} \cdot (C_{R,n} + C_{R-1,n} - 0.002) \Delta X \right] + 2\gamma (C_{R-1,n} - 3C_{R,n} + 0.002) \quad (15)$$

And the scheme is

$$\begin{aligned} & \left[ \gamma + \frac{C_{R,n+\frac{1}{2}}}{2} \cdot \Delta X \right] C_{R-1,n+1} + \\ & \left[ -3\gamma - \frac{2}{r} + \frac{C_{R,n+\frac{1}{2}}}{2} \cdot \Delta X \right] C_{R,n+1} \\ & = - \left[ \gamma + \frac{C_{R,n+\frac{1}{2}}}{2} \cdot \Delta X \right] C_{R-1,n} + \\ & \left[ 3\gamma - \frac{2}{r} - \frac{C_{R,n+\frac{1}{2}}}{2} \cdot \Delta X \right] C_{R,n} - \\ & 0.004 \left[ \gamma - \frac{C_{R,n+\frac{1}{2}}}{2} \cdot \Delta X \right] \end{aligned} \quad (16)$$

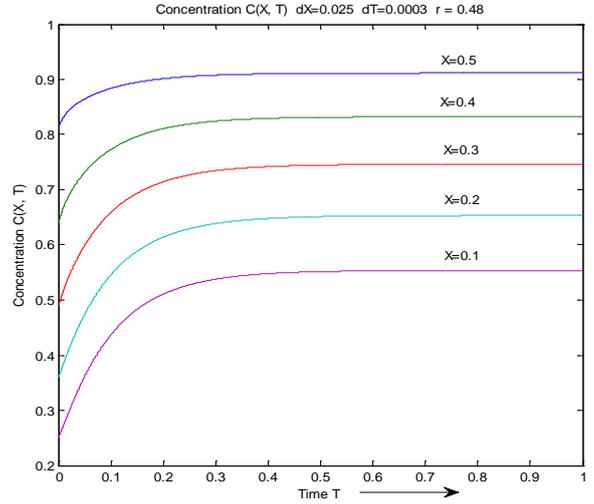


Figure 6: Graph of the Concentration  $C(X, T)$  vs. time  $T$  for  $r=0.48$

Table 1: Concentration  $C(X, T)$  Vs.  $X$  at time  $T > 0$

Concentration $C(X, T)$ $dX=0.025$ $dT=0.0003$ Ratio $r=0.48$					
$X \backslash T$	$T=0.03$	$T=0.06$	$T=0.09$	$T=0.12$	$T=0.15$
0	1	1	1	1	1
0.05	0.92765	0.93514	0.94009	0.94361	0.94618
0.10	0.85552	0.87311	0.88481	0.89315	0.89923
0.15	0.78202	0.80941	0.82778	0.84092	0.85050
0.20	0.70842	0.74482	0.76956	0.78732	0.80028
0.25	0.63589	0.68017	0.71074	0.73277	0.74889
0.30	0.56550	0.61623	0.65188	0.67770	0.69664
0.35	0.49814	0.55374	0.59352	0.62251	0.64383
0.40	0.43453	0.49332	0.53617	0.56760	0.59077
0.45	0.37517	0.43552	0.48026	0.51329	0.53772
0.50	0.32039	0.38072	0.42614	0.45988	0.48492
0.55	0.27036	0.32920	0.37405	0.40759	0.43256
0.60	0.22511	0.28107	0.32417	0.35659	0.38079
0.65	0.18452	0.23635	0.27654	0.30694	0.32971
0.70	0.14835	0.19487	0.23111	0.25866	0.27936
0.75	0.11622	0.15639	0.18775	0.21170	0.22973
0.80	0.08766	0.12054	0.14623	0.16591	0.18076
0.85	0.06207	0.08687	0.10623	0.12112	0.13237
0.90	0.03875	0.05484	0.06740	0.07708	0.08440
0.95	0.01691	0.02388	0.02933	0.03352	0.03670
1	0.001	0.001	0.001	0.001	0.001

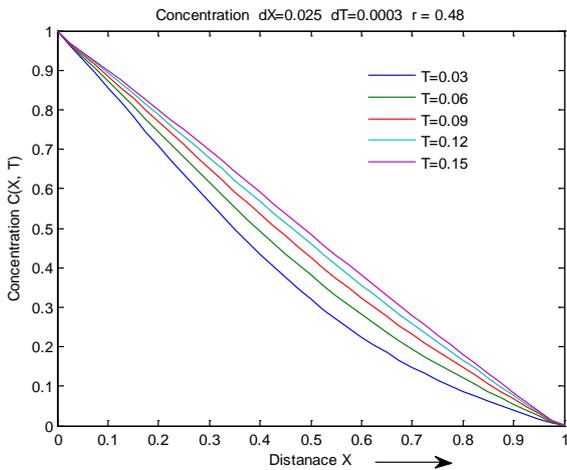


Figure 5: The Concentration  $C(X, T)$  vs. dist.  $X$  for  $r=0.48$

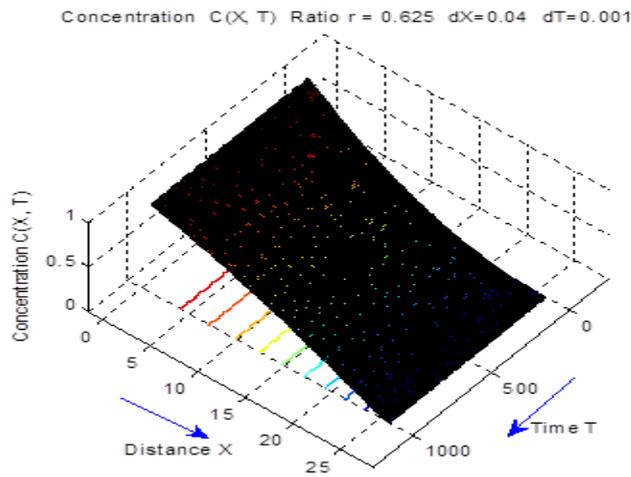


Figure 7: 3D behaviour of the Concentration  $C(X, T)$  versus distance  $X$  for different time  $T$ .

## CONCLUSIONS

The graphical and numerical solutions have been obtained to predict the possible concentration of a contaminated water in unsteady unidirectional seepage flow, through semi-finite, homogeneous, isotropic porous media subject to the source concentration that vary with the distance  $X$  and time  $T > 0$ . From the tabular values and graphs it is concluded that as distance  $X$  and time  $T$  increases the concentration of the contaminant water gradually decreases. The concentration  $C(X, T)$  of the contaminated water decreases as the distance  $X$  increases for the given time  $T > 0$ . Here the initial concentration of contaminated water at  $X=0$  is highest and it decreases as distance  $X$  increases for given time  $T > 0$ . It is physically fact that at the source the concentration of contaminated water is always highest and it is decreasing and dispersing from the source. It is also concluded from the graph (Fig. 6) of the concentration of contaminated water verses time  $T$  for given distance  $X$ , the concentration of contaminated water is increasing for small time  $T$  and then it becomes steady and constant as time  $T$  increases for given distances  $X=0.1, 0.2, 0.3, 0.4, 0.5$ . Hence, it is fact that at the initial source the dispersion of contaminated water is not fast, therefore the concentration of contaminated water is slightly increasing for small time  $T$ , for fixed distance  $X$  and then it becomes constant throughout the time for given distance  $X$ .

The numerical solution obtained here is immensely useful to control the intrusion of saline water before it contaminates the fresh water aquifer system; it is also helpful in making quantitative predictions on the possible contamination of groundwater supplies resulting from groundwater movement through buried wastes. The result is consistent with the physical phenomenon of longitudinal dispersion in miscible fluid flow through porous media.

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