

EFFECT OF CORRUGATION ANGLE ON THE HYDRODYNAMIC BEHAVIOUR OF POWER-LAW FLUIDS DURING A FLOW IN PLATE HEAT EXCHANGERS.

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ABSTRACT

In this study, CFD calculations were made in order to analyze the hydrodynamic behaviour of a power-law fluid in the channels of plate heat exchangers with corrugation angles of 30° and 60° during a non-isothermal flow.

For the observed laminar flow, the numerical results show that a typical velocity profile in the 3D channel of the plate heat exchanger with corrugation angle 30° assumes an approximate parabolic shape and that for a corrugation angle of 60° the profile have a irregular core.

Simulations considering and discarding the influence of temperature on the non-Newtonian fluid viscosity were performed for the two geometries and the impact of these variations on the relations between fanning friction factors and Reynolds number was analyzed as well as on the ratio between wall and bulk shear rate.

INTRODUCTION

Plate heat exchangers (PHE's) are commonly used on food industry for tasks including the high-temperature short-time pasteurization of milk, beer and fruit juices (Gut and Pinto, 2003) as well as the cooling of stirred yoghurt to stop lactic fermentation when a desired acidity is reached (Afonso et al., 2003).

Process food, pharmaceutical, chemical and biochemical fluid media generally have non-Newtonian characteristics and the shear thinning or thickening behaviour of these fluids greatly affects their thermal-hydraulic performance (Manglick and Ding, 1997).

PHE's offer several advantages like the low space requirement, efficiency, low fouling tendency, high

flexibility and easy maintenance. The design of the heat exchange equipment involve the determination of optimum pressure drops (Reppich, 1999) being referenced in the literature several studies directed to this important subject when leading with PHE's (Bassiouny and Martin, 1984; Delplace and Leuliet, 1995; Leuliet et al., 1990, 1987; Stasiak et al., 1996; Ciofalo et al., 1996; Wang and Sundén, 2003).

Computational fluid dynamics (CFD) calculations are also useful in order to understand the local properties of the flows in the complex geometries of PHE's being the geometries and flows more accurate described by implementing 3D channels (Kho and Müller-Steinhagen, 1999). Resorting to a 3D geometry Fernandes et al. (2005) made a non-isothermal, non-Newtonian analysis of stirred yoghurt cooling in a PHE, based on experimental data of Afonso et al. (2003), and a good agreement was found between numerical and experimental results.

The present study will be focused on numerical simulations of the hydrodynamic behaviour of a power-law fluid on the channels of commonly used PHE's.

NOMENCLATURE

a	parameter (-)
b	distance between plates (m)
C_p	specific heat ($\text{J kg}^{-1} \text{K}^{-1}$)
D_H	hydraulic diameter (m)
E	activation energy (J mol^{-1})
f	fanning friction factor (-)
k	thermal conductivity ($\text{W m}^{-1} \text{K}^{-1}$)
K_c	consistency index (Pa s^n)
L	effective length (m)

M_v volumetric flow rate ($\text{m}^3 \text{s}^{-1}$)
 n flow behaviour index (-)
 R ideal gas constant ($\text{J mol}^{-1} \text{K}^{-1}$)
 Re Reynolds number (-)
 T absolute temperature (K)
 u mean velocity (m s^{-1})
 w effective width (m)
 x spatial coordinate (m)
 y spatial coordinate (m)
 z spatial coordinate (m)

β corrugation angle ($^\circ$)
 ΔP pressure drop (Pa)
 ϕ area enlargement factor (-)
 $\dot{\gamma}$ shear rate (s^{-1})
 η apparent viscosity (Pa s)
 ρ density (kg m^{-3})

PROBLEM DESCRIPTION

In the present study is performed the simulation of a power-law fluid heating in different PHE's, being the corrugation angles, β , 30° and 60° .

Mathematical formulation

Mathematically, the problem was described by a set of equations that comprises the governing and constitutive equations. The problem could be divided in two problems of heat conduction in the plates and one of laminar non-isothermal flow inside the channel, being the governing equations the Fourier's law, to describe the heat conduction in the plates and the Navier-Stokes equations that include the conservation equations for mass, linear momentum and energy, to describe the flow.

The constitutive model used to describe the thermo-rheological behaviour of the fluid is:

$$\eta(T, \dot{\gamma}) = K_c \dot{\gamma}^{n-1} e^{\frac{E}{RT}} \quad (1)$$

where η is the apparent viscosity (Pa s), $\dot{\gamma}$ the shear rate (s^{-1}), T the absolute temperature (K), E the activation energy (J mol^{-1}), R the ideal gas constant ($R = 8.31451 \text{ J mol}^{-1} \text{K}^{-1}$), K_c the consistency index (Pa s^n) and n the flow behaviour index (-).



Fig. 1: Geometrical domain for PHE with $\beta = 60^\circ$.

The used rheological parameters were similar to the ones of a cloudy apple juice (Steffe, 1996): $K_c = 0.0499 \text{ Pa s}^n$, $E/R = 3065 \text{ K}$, $n = 0.5$.

Numerical resolution

The problem was numerically solved using the finite element method package POLYFLOW and the simulations were performed using a Dell Workstation PWS530 with 1GB of RAM. A complete description of numerical resolution of the present problem is provided by Fernandes et al. (2005).

Geometrical domain. Simulations were carried out in a 3D geometry constituted by three 3D elements: channel, inferior and superior plates. The construction method of the geometrical domain for a corrugation angle of 30° was based on available characteristics of a Pacetti RS 22 PHE (Fernandes et al., 2005). For $\beta = 60^\circ$, the method was the same varying only the corrugation angle, Fig. 1 and Fig.2.

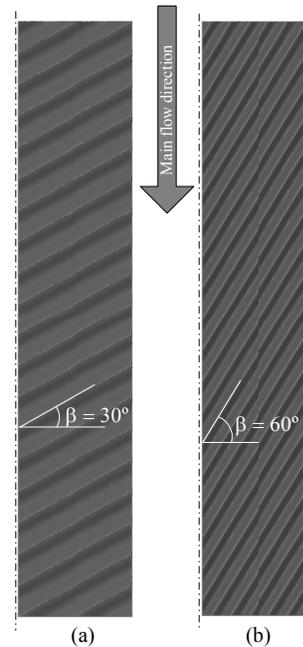


Fig. 2: View of half of a channel with (a) $\beta = 30^\circ$ and (b) $\beta = 60^\circ$. (----) Represents the symmetry axis.

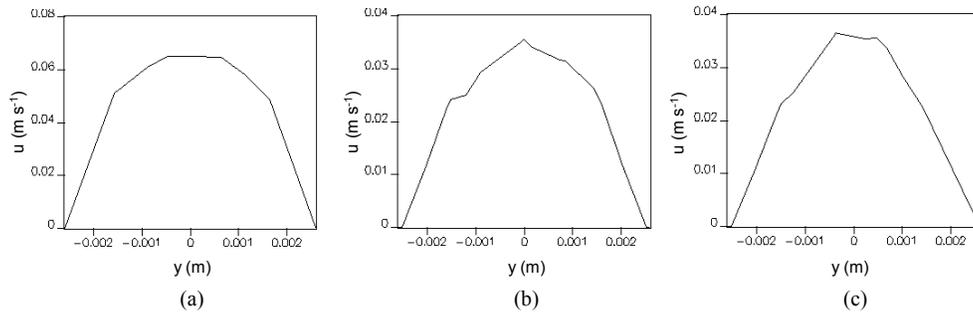


Fig. 3: Velocities profiles for $\beta = 30^\circ$ and flow rate of $3 \times 10^{-6} \text{ m}^3 \text{ s}^{-1}$ in the intersection of planes $z = 0.009$ and (a) $x = 0$; (b) $x = 0.09$ and (c) $x = 0.18$.

It was admitted that the heat exchanger had a parallel arrangement being consequently the flow simulations carried out in a single channel. Additionally, uniform flow was considered inside each channel and, for this reason, a symmetry axis was established, Fig. 2, simplifying the geometrical domain to half of a channel with length, L , of 0.19 m, width, w , of 0.036 m and plates distance, b , of 0.0026 m.

Boundary conditions. A fluid inlet temperature of 290 K was considered in all simulations with different flow rates, having the fluid a thermal conductivity, k , of $0.559 \text{ W m}^{-1} \text{ K}^{-1}$; specific heat, C_p , of $2\,935 \text{ J kg}^{-1} \text{ K}^{-1}$ and density, ρ , of $1\,050 \text{ kg m}^{-3}$ (Çengel, 1998).

A single boundary condition for the different flow rates in the form of a linear heat flux was established along the plates, adapting this way a counterflow general exponential expression to the closest boundary condition allowed by POLYFLOW (Fernandes et al., 2005). In all the simulations slip at the wall and heat losses to the surroundings were assumed to be non-existent.

RESULTS AND DISCUSSION

The plates with $\beta = 60^\circ$ and $\beta = 30^\circ$ have the same projected area but the effective area is higher on the former being obtained an area enlargement factor, ϕ (-), of 1.446

and 1.096 (Fernandes et al., 2005) for $\beta = 60^\circ$ and $\beta = 30^\circ$, respectively. The area enlargement factor was calculated by:

$$\phi = \frac{\text{Effective Area}}{\text{Projected Area}} \quad (2)$$

Analyzing the velocity field it was observed a laminar regime for all the simulated flow rates and the two different geometries. Due to complexity of the geometries the flow is 3D (Fernandes et al., 2005) being the typical velocity profiles along the channels approximately parabolic for $\beta = 30^\circ$, Fig. 3 (b) and (c) and presenting an irregular core for $\beta = 60^\circ$, Fig. 4 (b) and (c).

The observation of a laminar regime by analysis of velocity fields is on agreement with the obtained fanning friction factor, f (-), expressions, typical from laminar flows:

$$f = a \text{Re}^{-1} \quad (3)$$

The f -Re relations obtained when considering the influence of temperature on fluid viscosity were:

$$f = 19.549 \text{Re}^{-1} \quad (4)$$

$$f = 32.121 \text{Re}^{-1} \quad (5)$$

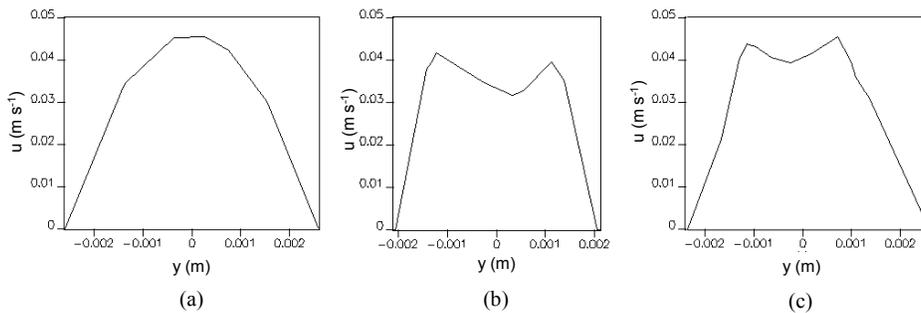


Figure 4: Velocities profiles for $\beta = 60^\circ$ and flow rate of $2 \times 10^{-6} \text{ m}^3 \text{ s}^{-1}$ in the intersection of planes $z = 0.009$ and (a) $x = 0$; (b) $x = 0.09$ and (c) $x = 0.18$.

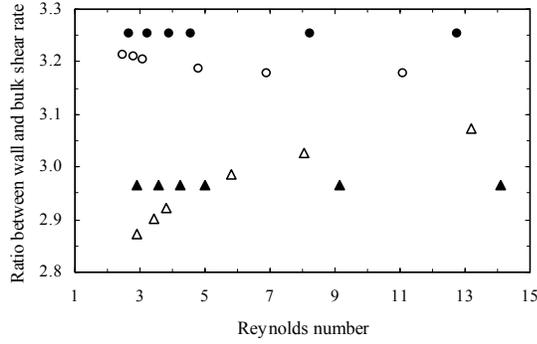


Fig. 5: Ratio between wall shear rate and bulk shear rate for $\beta = 30^\circ$ and $E/R = 3\ 065\ \text{K}$ (\circ); $\beta = 30^\circ$ and $E/R = 0\ \text{K}$ (\bullet); $\beta = 60^\circ$ and $E/R = 3\ 065\ \text{K}$ (Δ); $\beta = 60^\circ$ and $E/R = 0\ \text{K}$ (\blacktriangle).

for $\beta = 60^\circ$ and $\beta = 30^\circ$, respectively. Fanning friction factor and Reynolds number, Re (-), were mathematically expressed by:

$$f = \frac{\Delta P D_H}{2L\rho u^2}, \quad (6)$$

$$\text{Re} = \frac{\rho u D_H}{\eta} \quad (7)$$

where η is the apparent viscosity, i.e., the average viscosity on the complete channel calculated by POLYFLOW, and ΔP the pressure drop (Pa), also given by the numerical simulations. The hydraulic diameter, D_H (m), and the average velocity, u (m s^{-1}), were calculated according to the expressions:

$$D_H = 2b, \quad (8)$$

$$u = \frac{M_v}{wb}, \quad (9)$$

where M_v is the volumetric flow rate ($\text{m}^3 \text{s}^{-1}$).

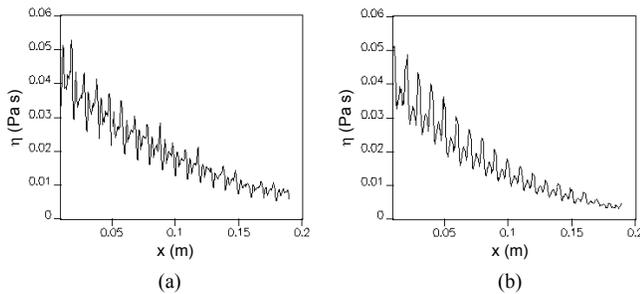


Fig. 6: Viscosity profiles along the PHE's channels with $E/R = 3\ 065\ \text{K}$ and for a flow rate of $2 \times 10^{-6} \text{ m}^3 \text{ s}^{-1}$ with (a) $\beta = 30^\circ$ and (b) $\beta = 60^\circ$.

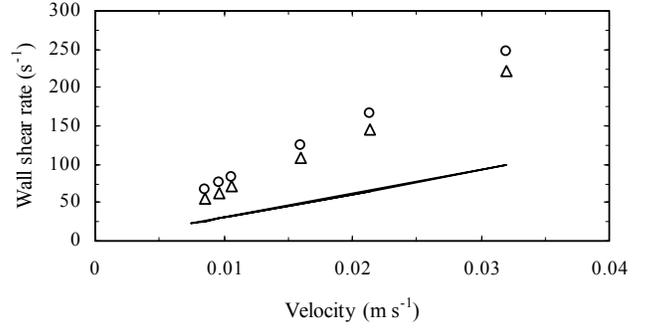


Fig. 7: Wall shear rate with $E/R = 3\ 065\ \text{K}$ for $\beta = 30^\circ$ (\circ), $\beta = 60^\circ$ (Δ). Wall shear rates for parallel plates ($-$) and $E/R = 0\ \text{K}$.

Simulations discarding the temperature effect on viscosity ($E/R = 0\ \text{K}$) were also performed and the obtained relations, for $\beta = 60^\circ$ and $\beta = 30^\circ$, respectively, took the form:

$$f = 38.394 \text{Re}^{-1} \quad (10)$$

$$f = 44.740 \text{Re}^{-1}. \quad (11)$$

Additionally, simulations with a Newtonian fluid, with the same physical properties of the present one, were performed. The obtained f - Re expressions for $\beta = 60^\circ$ and $\beta = 30^\circ$ were, respectively:

$$f = 58.206 \text{Re}^{-1} \quad (12)$$

$$f = 67.185 \text{Re}^{-1}. \quad (13)$$

It can be observed in the above equations that temperature effect on transversal and axial viscosity profiles affects the results of the f - Re relations. The constant a is higher for $\beta = 30^\circ$ on the different simulations and this fact is enhanced if the area enlargement factor is taking in account on the definition of the hydraulic diameter. In the case of the Newtonian fluid, if this geometrical parameter is included on the calculation of the hydraulic diameter (Kakaç and Liu, 2002), the constant a is 26.967 and 56.433 for $\beta = 60^\circ$ and $\beta = 30^\circ$, respectively.

On Fig. 5 it can be observed that the ratio between wall and bulk shear rates remains constant when $E/R = 0\ \text{K}$, for the different Reynolds numbers, but when the effect of the temperature on viscosity is considered, variations are observed and the referred ratio decrease and increase with the increase of Reynolds number for $\beta = 30^\circ$ and $\beta = 60^\circ$, respectively.

Shear rate presents an oscillatory behaviour in the studied complex geometries, conferring to viscosity the same behaviour, Fig. 6. It can be observed in Fig. 7 that for the same flow rate due to the higher wall shear rates developed for $\beta = 30^\circ$ viscosity presents higher variations,

Fig.6, i.e., the shear thinning effect is higher, being this effect escorted by the axial decrease of viscosity during the fluid heating along the channels.

On the processing of some food stuffs like the cooling of stirred yoghurt, where the viscosity can be undesirably and irreversibly reduced due to high shear rates, a PHE with $\beta = 60^\circ$ will be favourable, allowing the processing of higher flow rates, Fig. 7.

CONCLUSIONS

For the present power-law fluid and operating conditions a laminar flow was observed and an approximate parabolic velocity profile was obtained for a corrugation angle of 30° while for a corrugation angle of 60° the velocity profile presented an irregular core.

Simulations considering the influence of temperature on viscosity and discarding this effect conduced to different f - Re relations and different wall shear rates being the latter higher for a corrugation angle of 30° .

The constant of the f - Re expressions was higher for a corrugation angle of 30° , on the different simulations, and the shear thinning effect leded to a decrease of the constant a when comparing with a Newtonian fluid.

REFERENCES

- Afonso, I. M., Hes, L., Maia, J. M., and Melo, L. F., 2003. Heat transfer and rheology of stirred yoghurt during cooling in plate heat exchangers, *Journal of Food Engineering*, 57, pp. 179-187.
- Bassiouny, M. K., and Martin, H., 1984. Flow distribution and pressure drop in plate heat exchangers – I, *Chemical Engineering Science*, Vol. 39, No. 4, pp. 693-700.
- Çengel, Y. A., 1998, *Heat transfer: a practical approach*, WBC/McGraw-Hill Companies, International Edition.
- Ciofalo, M., Stasiek, J., and Collins, M. W., 1996. Investigation of flow and heat transfer in corrugated passages – II. Numerical simulation, *Int. J. Heat Mass Transfer*, 39, pp.165-192.
- Delplace, F., and Leuliet, J. C., 1995. Generalized Reynolds number for the flow of power law fluids in cylindrical ducts of arbitrary cross-section, *The Chemical Engineering Journal*, 56, pp. 33-37.
- Fernandes, C. S., Dias, R., Nóbrega, J. M., Afonso, I. M., Melo, L. F., and Maia, J. M., 2005. Simulation of stirred yoghurt processing in plate heat exchangers, *Journal of Food Engineering*, 69, pp. 281-290.
- Gut, J. A. W., and Pinto, J. M., 2003. Selecting optimal configurations for multisection plate heat exchangers in pasteurization processes, *Ind. Eng. Chem. Res.*, 42, pp. 6112-6124.
- Kho, T., and Müller-Steinhagen, 1999. An experimental and numerical investigation of plate heat transfer fouling and fluid flow in flat plate heat exchangers, *Trans IChemE*, Vol 77, Part A, pp. 124 – 130.
- Leuliet, J. C., Maigonnat, J. F., and Lalande, M., 1987. Etude de la perte de charge dans de échangeurs de chaleur à plaques traitant des produits non-newtoniens, *Rev. Gén. Therm. Fr.*, 308-309, pp. 445-450.
- Leuliet, J. C., Maigonnat, J. F., and Lalande, M., 1990. Écoulement et transferts de chaleur dans les échangeurs à plaques traitant des produits visqueux newtoniens et pseudoplastiques, *The Canadian Journal of Chemical Engineering*, 68, pp. 220-229.
- Manglick, R. M., and Ding, J., 1997. Laminar flow heat transfer to viscous power-law fluids in double-sine ducts, *International Journal of Heat and Mass Transfer*, Vol. 40, No. 6, pp. 1379-1390.
- Mehrabian, M. A., and Poulter, R., 2000. Hydrodynamics and thermal characteristics of corrugated channels: computacional approach, *Applied Mathematical Modelling*, 24, pp. 343-364.
- Reppich, M., 1999. Use of high performance plate heat exchangers in chemical and process industries, *Int. J. Therm. Sci.*, 38, pp. 999-1008.
- Stasiek, J., Collins, M. W., Ciofalo, M., and Chew, P. E., 1996. Investigation of flow and heat transfer in corrugated passages – I, Experimental results, *Int. J. Heat Mass Transfer*, 39, pp. 149-164.
- Steffe, J. F., 1996, *Rheological methods in food process engineering*, 2nd ed. , Freeman Press, East Lansing.
- Wang, L., and Sundén, B., 2003. Optimal design of plate heat exchangers with and without pressure drop specifications, *Applied Thermal Engineering*, 23, pp. 295-311.