STOKES FLOW PAST A TWO-LAYERS HETEROGENEOUS POROUS SPHERE WITH THE EFFECT OF STRESS JUMP CONDITION: AN EXACT SOLUTION

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The flow past a porous sphere has extensive industrial and engineering applications, such as the flow of pulverized coals particulate during combustion, sedimentation of fine particulate suspensions, flow in porous beds etc. Several studies have been done about the flow past a porous body under different models and boundary conditions. The models used in their investigations can be divided into the following categories: (i) Adopting Darcy equation to describe the flow in porous media and Stokes equations to describe the flow in the free fluid with the continuity conditions of velocity and pressure at the interface or the Beavers-Joseph (BJ) interface conditions; (ii) Adopting the Darcy-Brinkman equation to describe the flow inside the porous region and Stokes equations to describe the flow in the free fluid region with interface conditions which are the continuity of velocity components and stresses at the interface or with the Ochoa-Tapia and Whitaker interface conditions, in which shearing stress jump at the interface is considered. However, most of researches just considered one-layer porous medium whether it was a porous sphere or a porous sphere containing a solid concentric spherical core or a concentric spherical cavity. P. D. Verma and B. S. Bhatt (1976) investigated the flow past a heterogeneous porous sphere with the Darcy model.

In this paper, a heterogeneous porous sphere containing two-layers porous medium of internal radius ‘R1’ and external radius ‘R2’ is considered, which is immersed in a uniform stream of velocity v. The internal porous region, the external porous region and the free fluid region are denoted by regions I, II and III respectively. Darcy-Brinkman equation is adopted to describe the flow in region I and region II separately and Stokes equations are adopted to describe the flow in region III. The continuity of the velocity components and stresses are taken at the interface between region I and region II. The continuity of the velocity components and normal stress and shearing stress jump conditions are taken at the interface between region II and region III. The exact flow solutions are derived. The expressions of the drag force on the surface of the sphere and the stream function for free fluid region, the internal porous region and the external porous region are determined. Moreover, the analytical solutions have been verified in some limiting cases. If the permeability in external region reaches zero, the current expression of drag will be identical with the expression for stokes flow past a solid sphere. If the internal radius R1 approaches zero or the internal and external porous medium have the same physical properties, the current solutions will agree with the results of A. C. Srivastava and Neetu (2005) for flow past a homogeneous porous sphere. Furthermore, if the stress jump coefficient is zero and the permeability is small, the current expression of the drag on the sphere is identical with the expression deduced by Yu and Kaloni(1988). For the same reason, if the permeability in internal region approaches zero, the current solutions will agree with A. C. Srivastava and Neetu (2006) for stokes flow past a porous sphere with a solid core. If the permeability in the internal region reaches infinity, the current solutions agree with the results for flow past a porous spherical shell with a concentric spherical cavity under the continuity conditions of velocity components and stresses at the interface between porous shell and internal cavity. In addition, it is found that both the permeability and the stress jump coefficient have significant effects on the drag. The variation of drag with the permeability in internal and external regions for different inner to outer ratio (R1/R2) as well as for different stress jump coefficient is discussed. It is found that the drag increases with the decrease of the internal and external permeability and the stress jump coefficient.