Prediction of complex systems using Grey Models

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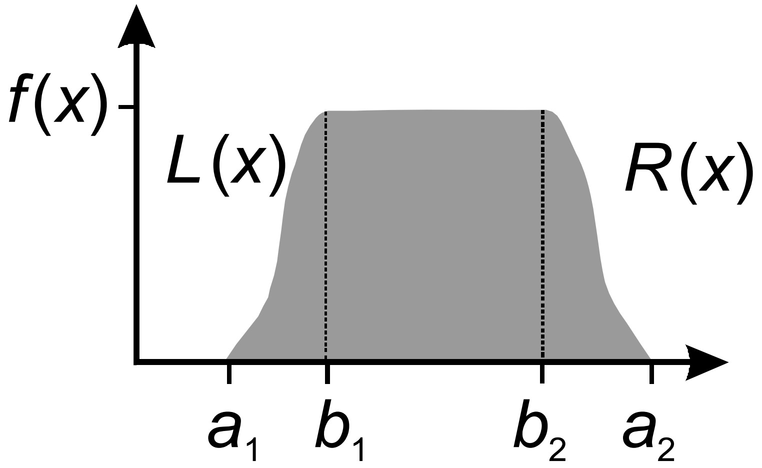
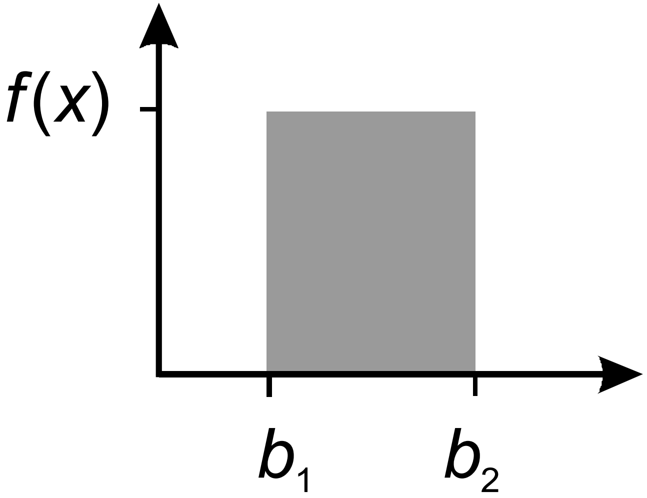
Mathematical methods for Indetermination

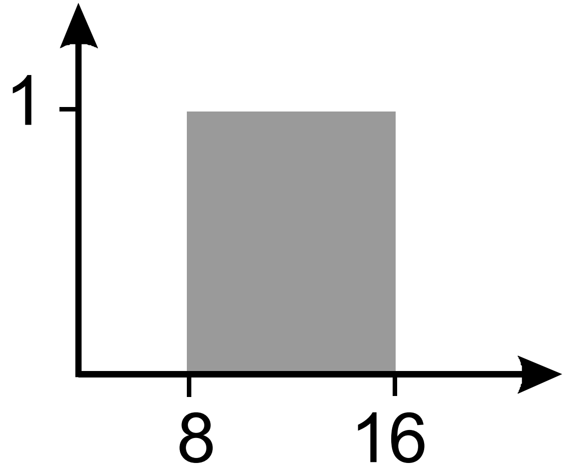
- Statistics and Probability
- Fuzzy-Sets
- Grey Systems
- Rough Sets
- Expert Systems
- Neuronal Networks
- Genetic Procedures
- Shannon's Entropy
- Chaos Theory

more than 300 Prediction theories

Grey Numbers



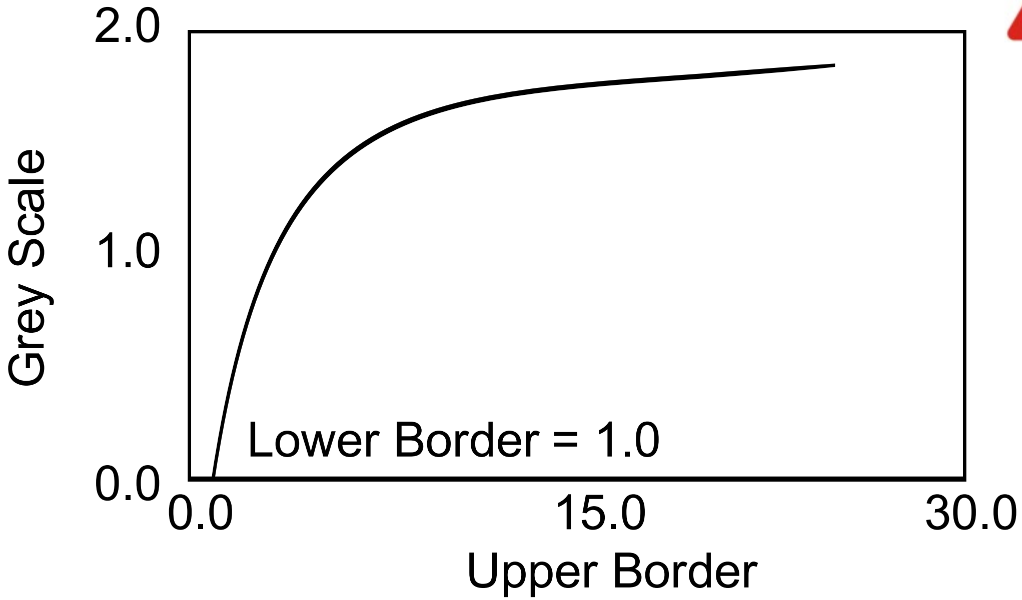




Other Measures of Deviation

Grey Scale

$$g^0 = \frac{2 \cdot |b_2 - b_1|}{b_2 + b_1}$$



Relative Grey Scale

$$g^0 = \left(\frac{2 \cdot |b_2 - b_1|}{b_2 + b_1} + \max\left\{ \frac{|b_1 - a_1|}{b_1}, \frac{|b_2 - a_2|}{b_2} \right\} \right)$$

Topics of Grey Models

- Grey Decision Making
- Grey Forecasting Models
- Grey Linear Programming

Grey Forecasting:

Original measured Series

$$X^{(0)} = \begin{pmatrix} x^{(0)}(1) \\ x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(4) \end{pmatrix}$$

Accumulated Series

$$X^{(1)}(m) = \sum_{n=1}^{m} x^{(0)}(n); \, m = 1 \ldots k$$

Forecast Model

$$x^{(1)}(t) \rightarrow z^{(1)}(k) = 0.5 \cdot x^{(1)}(k) + 0.5 \cdot x^{(1)}(k-1)$$

$$\frac{dx^{(1)}}{dt} + a \cdot x^{(1)}(t) = b$$

Assumption about behavior!

$$\frac{dx^{(1)}}{dt} \rightarrow x^{(1)}(k+1) - x^{(1)}(k) = x^{(0)}(k+1)$$

$$x^{(1)}(k+1) = \left(x^{(0)}(1) - \frac{b}{a} \right) \cdot e^{-a \cdot k} + \frac{b}{a}$$

Grey Exponential Model

Forecast Model

Grey Exponential Model: GM(1,1)

Grey Verhulst Model: GM(2,1)

Polyfactor Grey Model: GM(1,N)

Polynomial Grey Model: GM(0,N)

Grey Fuzzy Model: GFM(1,1),

ARIMA and Grey Model: GDM(2,2,1),

Deterministic Grey Dynamic Model: DGDM(1,1,1)

Adv. Deterministic Grey Dynamic Model: DGDMMI(1,1,1)

Unequal Interval Revised Model: UIRGM(1,1)

Example I.

Sturm surge in the Dutch North Sea

Regional Frequency Analysis
(five locations of about 100 years extreme water events)

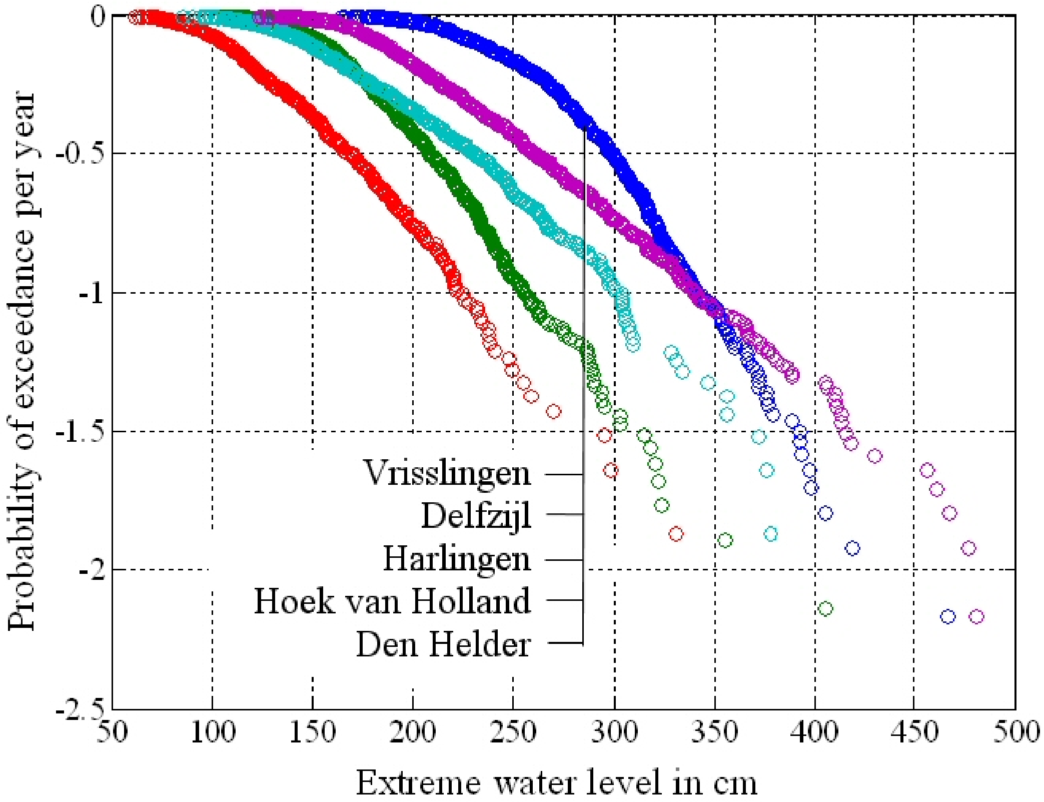
Using as one population (increasing sample size)

Goal:

Estimating extreme value with return period 10^{-4} Years

Example I.

Sturm surge in the Dutch North Sea



Example I.

- Using Grey Exponential Model
- Using Grey Verhulst Model

Water level height for 10^{-4} Probability of Exceedance

Location	Basispeilen	Exponential	Grey exponential
Delfzijl	613	705	720
Harlingen	501	620	635
Hoek	500	470	550
Vlissingen	545	560	610
Den Helder	441	460	500

Correlation

Change of System

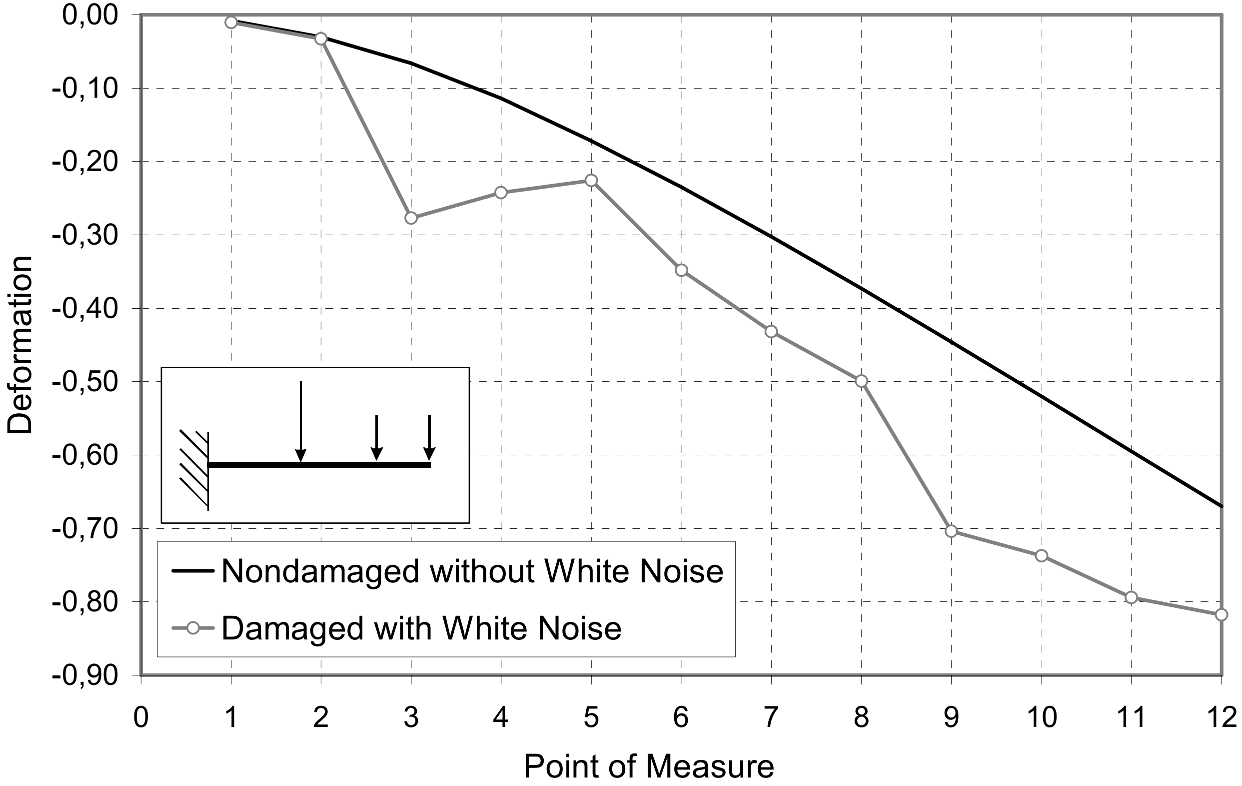
$$X_1 = \begin{pmatrix} |x_1(1) - x_0(1)| \\ |x_1(2) - x_0(2)| \\ |x_1(3) - x_0(3)| \\ \dots \\ |x_1(4) - x_0(4)| \end{pmatrix}$$

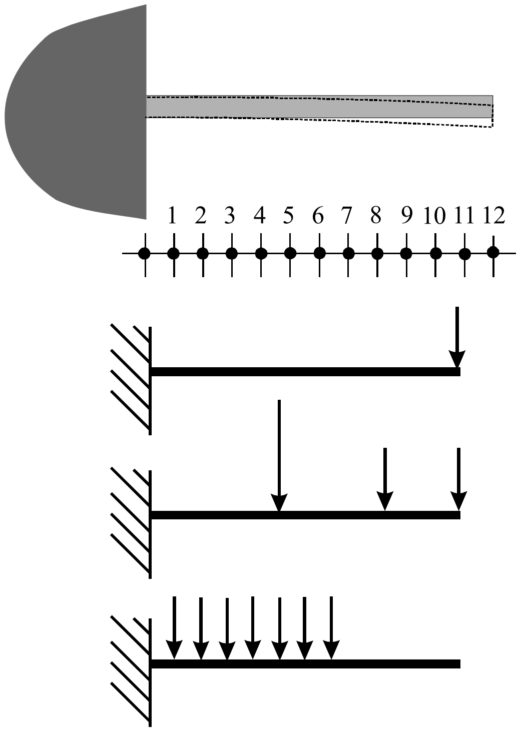
$$X_i = \begin{pmatrix} |x_i(1) - x_{i-1}(1)| \\ |x_i(2) - x_{i-1}(2)| \\ |x_i(3) - x_{i-1}(3)| \\ \dots \\ |x_i(4) - x_{i-1}(4)| \end{pmatrix}$$

$$\xi_i(k) = \frac{\min_i \min_k X + \alpha \max_i \max_k X}{X + \max_i \max_k X} \qquad 0 \le \alpha \le 1$$

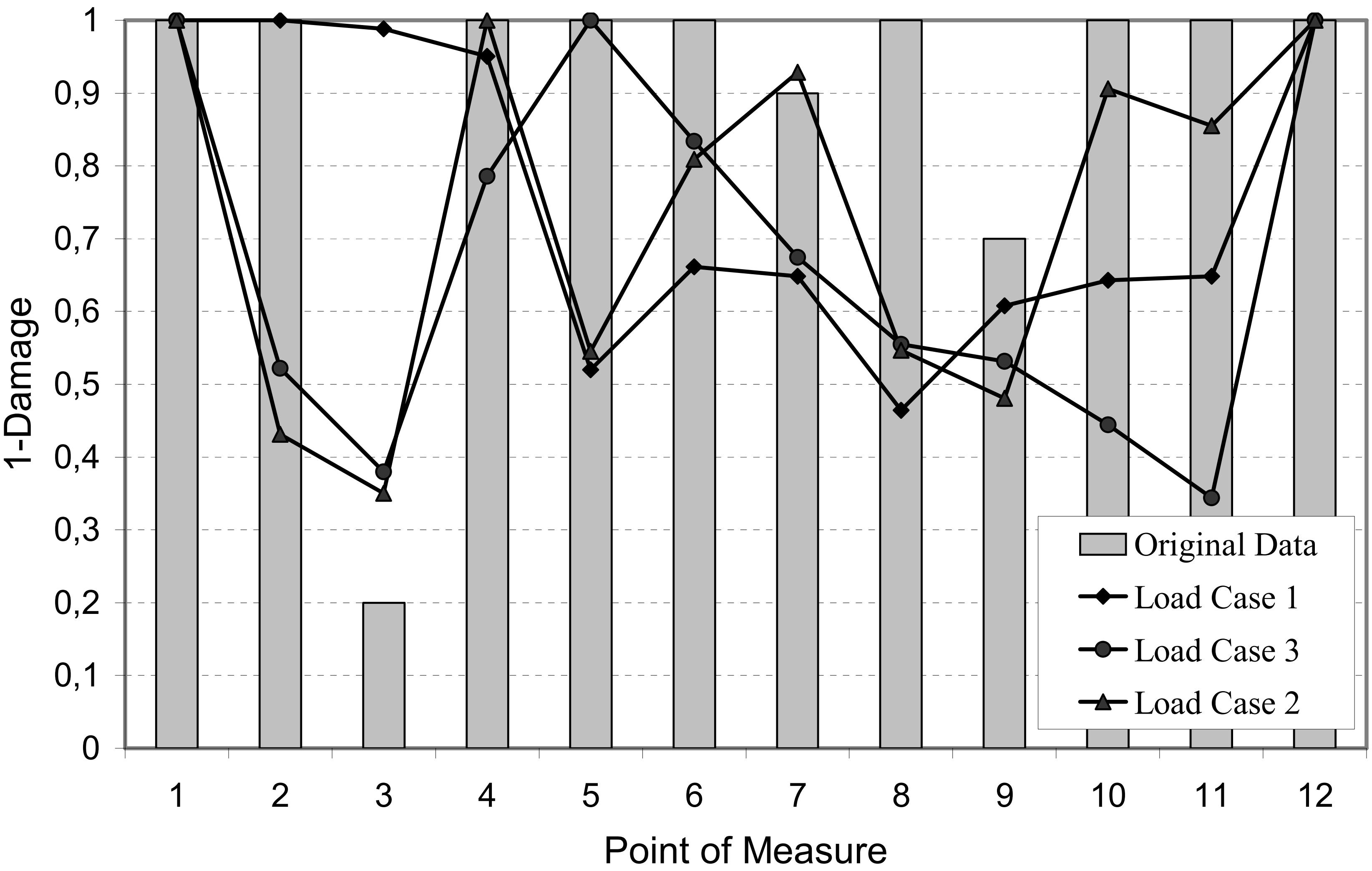
Example II.

Change of System





Example II.



1
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
1-Damage
1
2
3
4
5
6
7
8
9
10
11
12
Point of Measure
Original Data
Load Case 1
Load Case 3
Load Case 2

Conclusion

Grey Systems is another method of considering indetermination

In some cases it gives better results.

Recommendation for application requires further research