TIME FRACTIONAL TRANSPORT MODEL FOR FLOW THROUGH TIGHT POROUS MEDIA

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Modelling the flow of fluid through tight porous media, such as unconventional hydrocarbon reservoirs, is very challenging and it is a growing sector and must be addressed. Shale gas is found in such tight porous rocks which are characterized by nano-scale size porous networks with ultra-low permeability [1,2].

Here gas non-linear transport models for reservoir simulations of single-phase gas through homogeneous tight rocks, [3], is combined with a fractional calculus method, [4,5], to pose a new time-fractional advection-diffusion transport model [6,7] for the pressure field, $p = p(x,t)$,

$$\frac{\partial^{\propto} p}{\partial t^{\propto}} = \mathcal{L}(p) + R(x,t), \qquad t > 0, \quad a \le x \le b; \;\; 0 < \propto < 1,$$

and with suitable initial and boundary conditions, where $\mathcal{L}(\cdot) \equiv \frac{\partial}{\partial x}\left(-U(\cdot) + K\frac{\partial}{\partial x}(\cdot)\right)$ is a second order differential operator, $\frac{\partial^{\propto}}{\partial t^{\propto}}$ is the Caputo fractional derivative or order $\propto$, R is an source term, and $b - a$ is the length of the reservoir. In these models, the apparent diffusivity is $K = K(x;p)$, and the apparent convective velocity is $U = U(x;\, p, p_x)$; thus either or both of K and U can be non-linear. We will analyse the solutions for different fractional order $\propto$, and compare the solutions with other models and against data where available.

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