EFFECT OF CORRUGATION ANGLE ON THE HYDRODYNAMIC BEHAVIOUR OF POWER-LAW FLUIDS DURING A FLOW IN PLATE HEAT EXCHANGERS.


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ABSTRACT

In this study, CFD calculations were made in order to analyze the hydrodynamic behaviour of a power-law fluid in the channels of plate heat exchangers with corrugation angles of 30º and 60º during a non-isothermal flow.

For the observed laminar flow, the numerical results show that a typical velocity profile in the 3D channel of the plate heat exchanger with corrugation angle 30º assumes an approximate parabolic shape and that for a corrugation angle of 60º the profile have a irregular core.

Simulations considering and discarding the influence of temperature on the non-Newtonian fluid viscosity were performed for the two geometries and the impact of these variations on the relations between fanning friction factors and Reynolds number was analyzed as well as on the ratio between wall and bulk shear rate.

INTRODUCTION

Plate heat exchangers (PHE’s) are commonly used on food industry for tasks including the high-temperature short-time pasteurization of milk, beer and fruit juices (Gut and Pinto, 2003) as well as the cooling of stirred yoghurt to stop lactic fermentation when a desired acidity is reached (Afonso et al., 2003).

Process food, pharmaceutical, chemical and biochemical fluid media generally have non-Newtonian characteristics and the shear thinning or thickening behaviour of these fluids greatly affects their thermal-hydraulic performance (Manglick and Ding, 1997).

PHE’s offer several advantages like the low space requirement, efficiency, low fouling tendency, high flexibility and easy maintenance. The design of the heat exchange equipment involve the determination of optimum pressure drops (Reppich, 1999) being referenced in the literature several studies directed to this important subject when leading with PHE’s (Bassiouny and Martin, 1984; Delplace and Leuliet, 1995; Leuliet et al., 1990, 1987; Stasiek et al., 1996; Ciofalo et al., 1996; Wang and Sundén, 2003).

Computational fluid dynamics (CFD) calculations are also useful in order to understand the local properties of the flows in the complex geometries of PHE’s being the geometries and flows more accurate described by implementing 3D channels (Kho and Müller-Steinhagen, 1999). Resorting to a 3D geometry Fernandes et al. (2005) made a non-isothermal, non-Newtonian analysis of stirred yoghurt cooling in a PHE, based on experimental data of Afonso et al. (2003), and a good agreement was found between numerical and experimental results.

The present study will be focused on numerical simulations of the hydrodynamic behaviour of a power-law fluid on the channels of commonly used PHE’s.

NOMENCLATURE

a parameter (-)
b distance between plates (m)
Cp specific heat (J kg⁻¹ K⁻¹)
Dh hydraulic diameter (m)
E activation energy (J mol⁻¹)
f fanning friction factor (-)
k thermal conductivity (W m⁻¹ K⁻¹)
Kc consistency index (Pa s⁰)
L effective length (m)
**PROBLEM DESCRIPTION**

In the present study is performed the simulation of a power-law fluid heating in different PHE’s, being the corrugation angles, $\beta$, $30^\circ$ and $60^\circ$.

**Mathematical formulation**

Mathematically, the problem was described by a set of equations that comprises the governing and constitutive equations. The problem could be divided in two problems of heat conduction in the plates and one of laminar non-isothermal flow inside the channel, being the governing equations the Fourier’s law, to describe the heat conduction in the plates and the Navier-Stokes equations that include the conservation equations for mass, linear momentum and energy, to describe the flow.

The constitutive model used to describe the thermo-rheological behaviour of the fluid is:

$$\eta(T, \dot{\gamma}) = K_c \dot{\gamma}^{n-1} e^{E/R_T}$$  \hspace{1cm} (1)

where $\eta$ is the apparent viscosity (Pa s), $\dot{\gamma}$ the shear rate (s$^{-1}$), $T$ the absolute temperature (K), $E$ the activation energy (J mol$^{-1}$), $R$ the ideal gas constant ($R = 8.31451$ J mol$^{-1}$ K$^{-1}$), $K_c$ the consistency index (Pa s$^n$) and $n$ the flow behaviour index (-).

The used rheological parameters were similar to the ones of a cloudy apple juice (Steffe, 1996): $K_c = 0.0499$ Pa s$^n$, $E/R = 3.065$ K., $n = 0.5$.

**Numerical resolution**

The problem was numerically solved using the finite element method package POLYFLOW and the simulations were performed using a Dell Workstation PWS530 with 1GB of RAM. A complete description of numerical resolution of the present problem is provided by Fernandes et al. (2005).

**Geometrical domain.** Simulations were carried out in a 3D geometry constituted by three 3D elements: channel, inferior and superior plates. The construction method of the geometrical domain for a corrugation angle of $30^\circ$ was based on available characteristics of a Pacetti RS 22 PHE (Fernandes et al., 2005). For $\beta = 60^\circ$, the method was the same varying only the corrugation angle, Fig. 1 and Fig.2.
Fig. 3: Velocities profiles for $\beta = 30^\circ$ and flow rate of $3 \times 10^{-6}$ m$^3$ s$^{-1}$ in the intersection of planes $z = 0.009$ and (a) $x = 0$; (b) $x = 0.09$ and (c) $x = 0.18$.

It was admitted that the heat exchanger had a parallel arrangement being consequently the flow simulations carried out in a single channel. Additionally, uniform flow was considered inside each channel and, for this reason, a symmetry axis was established, Fig. 2, simplifying the geometrical domain to half of a channel with length, $L$, of 0.19 m, width, $w$, of 0.036 m and plates distance, $b$, of 0.0026 m.

**Boundary conditions.** A fluid inlet temperature of 290 K was considered in all simulations with different flow rates, having the fluid a thermal conductivity, $k$, of 0.559 W m$^{-1}$ K$^{-1}$; specific heat, $C_p$, of 2 935 J kg$^{-1}$ K$^{-1}$ and density, $\rho$, of 1 050 kg m$^{-3}$ (Çengel, 1998).

A single boundary condition for the different flow rates in the form of a linear heat flux was established along the plates, adapting this way a counterflow general exponential expression to the closest boundary condition allowed by POLYFLOW (Fernandes et al., 2005). In all the simulations slip at the wall and heat losses to the surroundings were assumed to be non-existent.

**RESULTS AND DISCUSSION**

The plates with $\beta = 60^\circ$ and $\beta = 30^\circ$ have the same projected area but the effective area is higher on the former being obtained an area enlargement factor, $\phi$ (-), of 1.446 and 1.096 (Fernandes et al., 2005) for $\beta = 60^\circ$ and $\beta = 30^\circ$, respectively. The area enlargement factor was calculated by:

$$\phi = \frac{\text{Effective Area}}{\text{Projected Area}} .$$

Analyzing the velocity field it was observed a laminar regime for all the simulated flow rates and the two different geometries. Due to complexity of the geometries the flow is 3D (Fernandes et al., 2005) being the typical velocity profiles along the channels approximately parabolic for $\beta = 30^\circ$, Fig. 3 (b) and (c) and presenting an irregular core for $\beta = 60^\circ$, Fig. 4 (b) and (c).

The observation of a laminar regime by analysis of velocity fields is on agreement with the obtained fanning friction factor, $f$ (-), expressions, typical from laminar flows:

$$f = a \, R e^{-1} .$$

The $f$-Re relations obtained when considering the influence of temperature on fluid viscosity were:

$$f = 19.549 \, R e^{-1}$$

$$f = 32.121 \, R e^{-1}$$

Figure 4: Velocities profiles for $\beta = 60^\circ$ and flow rate of $2 \times 10^{-6}$ m$^3$ s$^{-1}$ in the intersection of planes $z = 0.009$ and (a) $x = 0$; (b) $x = 0.09$ and (c) $x = 0.18$. 
Fig. 5: Ratio between wall shear rate and bulk shear rate for $\beta = 30^\circ$ and $E/R = 3\,065$ K (•); $\beta = 30^\circ$ and $E/R = 0$ K (●); $\beta = 60^\circ$ and $E/R = 3\,065$ K (∆); $\beta = 60^\circ$ and $E/R = 0$ K (▲).

for $\beta = 60^\circ$ and $\beta = 30^\circ$, respectively. Fanning friction factor and Reynolds number, $\text{Re} (\cdot)$, were mathematically expressed by:

$$ f = \frac{\Delta P D_H}{2L \rho u^2}, \quad (6) $$

$$ \text{Re} = \frac{\rho u D_H}{\eta}, \quad (7) $$

where $\eta$ is the apparent viscosity, i.e., the average viscosity on the complete channel calculated by POLYFLOW, and $\Delta P$ the pressure drop (Pa), also given by the numerical simulations. The hydraulic diameter, $D_H$ (m), and the average velocity, $u$ (m s$^{-1}$), were calculated according to the expressions:

$$ D_H = 2b, \quad (8) $$

$$ u = \frac{M_v}{\rho b}, \quad (9) $$

where $M_v$ is the volumetric flow rate (m$^3$ s$^{-1}$).

Simulations discarding the temperature effect on viscosity ($E/R = 0$ K) were also performed and the obtained relations, for $\beta = 60^\circ$ and $\beta = 30^\circ$, respectively, took the form:

$$ f = 38.394 \, \text{Re}^{-1} \quad (10) $$

$$ f = 44.740 \, \text{Re}^{-1}. \quad (11) $$

Additionally, simulations with a Newtonian fluid, with the same physical properties of the present one, were performed. The obtained $f$ - $\text{Re}$ expressions for $\beta = 60^\circ$ and $\beta = 30^\circ$ were, respectively:

$$ f = 58.206 \, \text{Re}^{-1} \quad (12) $$

$$ f = 67.185 \, \text{Re}^{-1}. \quad (13) $$

It can be observed in the above equations that temperature effect on transversal and axial viscosity profiles affects the results of the $f$ - $\text{Re}$ relations. The constant $a$ is higher for $\beta = 30^\circ$ on the different simulations and this fact is enhanced if the area enlargement factor is taking in account on the definition of the hydraulic diameter. In the case of the Newtonian fluid, if this geometrical parameter is included on the calculation of the hydraulic diameter (Kakaç and Liu, 2002), the constant $a$ is 26.967 and 56.433 for $\beta = 60^\circ$ and $\beta = 30^\circ$, respectively.

On Fig. 5 it can be observed that the ratio between wall and bulk shear rates remains constant when $E/R = 0$ K, for the different Reynolds numbers, but when the effect of the temperature on viscosity is considered, variations are observed and the referred ratio decrease and increase with the increase of Reynolds number for $\beta = 30^\circ$ and $\beta = 60^\circ$, respectively.

Shear rate presents an oscillatory behaviour in the studied complex geometries, conferring to viscosity the same behaviour, Fig. 6. It can be observed in Fig. 7 that for the same flow rate due to the higher wall shear rates developed for $\beta = 30^\circ$ viscosity presents higher variations,
Fig. 6, i.e., the shear thinning effect is higher, being this effect escorted by the axial decrease of viscosity during the fluid heating along the channels.

On the processing of some food stuffs like the cooling of stirred yoghurt, where the viscosity can be undesirably and irreversibly reduced due to high shear rates, a PHE with $\beta = 60^\circ$ will be favourable, allowing the processing of higher flow rates, Fig. 7.

CONCLUSIONS

For the present power-law fluid and operating conditions a laminar flow was observed and an approximate parabolic velocity profile was obtained for a corrugation angle of $30^\circ$ while for a corrugation angle of $60^\circ$ the velocity profile presented an irregular core.

Simulations considering the influence of temperature on viscosity and discarding this effect conduced to different $f$ - $Re$ relations and different wall shear rates being the latter higher for a corrugation angle of $30^\circ$.

The constant of the $f$ - $Re$ expressions was higher for a corrugation angle of $30^\circ$, on the different simulations, and the shear thinning effect leaded to a decrease of the constant $a$ when comparing with a Newtonian fluid.

REFERENCES


